Wind-Based Robust Trajectory Optimization using Meteorological Ensemble Probabilistic Forecasts

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Uncertainty and 4D trajectories

\[ t_0 + p(x) = t_0 + t_1 + t_2 \]

Weather? Mass? Performance? ...

\[ p(x) \]

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TBO-Met project

Pre-Tactical Phase up to 3 Hours before departure
- ECMWF EPS forecasts
- Robust Trajectory Planning
- Convection Probability fields
- Wind Uncertainty fields
- SBT

Tactical Phase (during execution)
- Nowcasts
- Storm avoidance
- Agreed RBT
- Executed RBT
- Revised RBT

T -6Hours  T -3Hours  T (departure)  E- 1/2 to 3Hours  E (event)
Research scope

ATM planning phases

- Strategic Long Term (Months)
- Strategic Mid Term (days)
- Pre-Tactical Short Term (hours)
- Tactical Short Term (minutes)
- Execution (real time)

Trajectory Status

- Business Developed Trajectory (BDT)
- Shared Business Trajectory (SBT)
- Reference Business Trajectory (RBT)
- Revised & Updated Business Trajectory

Planning
- Airspace User (Airline)

Sharing
- Network Manager (Eurocontrol)

Agreeing

Revising
- ATC & Aircraft

Updating

Trajectory Coordinator
4D Trajectory Planning
Contributions of this work

- Developing a methodology for 4D flight plan optimization under uncertainty
- Incorporate predictability into this methodology as an objective
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Why use ensemble forecasting?

Weather forecasts are uncertain, because nonlinear, complex, chaotic dynamics amplify:

1. Uncertainty in initial conditions
2. Physical modelling/parametrization errors
3. Numerical error and computational limitations

Deterministic forecasts have limited usefulness
Why use ensemble forecasting?

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Ensemble Prediction Systems

Figure: Uncertainty propagation
Figure: A subset of the PEARP ensemble forecast at 250 hPa
The 3-DoF Point-Mass Model

3 Position states:
- Latitude ($\phi$)
- Longitude ($\lambda$)
- Altitude ($z$)

1 Additional state:
- Mass ($m$)

3 Velocity states:
- Airspeed ($v$)
- Heading ($\psi$)
- Flight Path Angle ($\gamma$)

3 Controls:
- Throttle ($\pi$)
- Lift Coeff. ($C_L$)
- Bank Angle ($\mu$)

\[ C_L = C_L(\alpha) \]
Phases of a flight

- Takeoff
- Initial climb
- Cruise
- Descent
- Approach
- & Landing

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Dynamic model for cruise

Cruise dynamical system

\[
\dot{\phi} = \frac{v \cdot \cos(\psi) + w_y(\phi, \lambda, t)}{R_M + z}
\]

\[
\dot{\lambda} = \frac{v \cdot \sin(\psi) + w_x(\phi, \lambda, t)}{(R_M + z) \cos(\phi)}
\]

\[
\dot{v} = m^{-1}(\text{Thrust}(\pi, v, \rho, T) - \text{Drag}(v, m, \rho, T))
\]

\[
\dot{m} = -\text{Fuel Consumption}(\pi, v, \rho, T)
\]

- Forces, constraints & fuel consumption from BADA model
Wind impact on the dynamics

- True Airspeed (TAS)
- Heading
- Wind speed
- Ground speed
The Optimal Control Problem

**OCP**

\[ x \in \mathbb{R}^n, \ u \in \mathbb{R}^m, \ g \in \mathbb{R}^{ng} \]

\[
\min J = \varphi(x_0, x_f, t_0, t_f) + \int_{t_0}^{t_f} \mathcal{L}(x, u, t) dt
\]

subject to:

\[
\frac{dx}{dt} = f(x, u, t)
\]

\[
g_L \leq g(x, u, t) \leq g_U
\]

\[
\ b_L \leq b(x_0, x_f, t_0, t_f) \leq b_U
\]

- **Common cost functional in ATM:** \( J = -m_f + CI \cdot t_f \)
Direct collocation methods

NLP

\[
\min J = \varphi(x_0, x_M, t_0, t_M) + (t_f - t_0) \sum_{k=0}^{M} w_k L(x_k, u_k, t_k)
\]

subject to:

\[
D_k(x_0, \ldots, x_N) = f(x_k, u_k, t_k), \quad k \in \{0, \ldots, M\}
\]

\[
g_L \leq g(x_k, u_k, t_k) \leq g_U, \quad k \in \{0, \ldots, M\}
\]

\[
b_L \leq b(x_0, x_M, t_0, t_M) \leq b_U
\]
The Dynamical System with Uncertainty

\[ \frac{dx}{dt} = f(x, u, t, p) \]

- Uncertainty is represented by a random variable \( p \in \mathbb{R}^{n_p} \)
- Note that the uncertainty \( p \) is static (i.e. this is not a SDE)
The robust OCP

- Discretize $p$ with a Polynomial Chaos rule
- Approximate expectation with $\mathbb{E}(h(p)) = \sum_{i=0}^{N} w_i h(p_i)$
- Create a copy of the state $x_i$ for each $p_i$ and average the cost functional
- Problem size is $\mathcal{O}(nN)$

Dynamical system for the open-loop ROCP

$$\frac{dx_i}{dt} = f(x_i, u, t, p_i), \quad i \in \{0, \ldots, N\}$$
The robust OCP II

- Declare $q \leq m$ states to be tracked
- Make up to $q$ controls scenario-specific

Dynamical system for the state-tracking ROCP

\[ \frac{dx_i}{dt} = f(x_i, u, t, p_i), \quad i \in \{0, \ldots, N\} \]

\[ T(x_i - x_j) = 0 \]

where $T \in \mathbb{R}^{n \times q}$ is the (full-rank) tracking matrix
Applying the ST-ROCP formulation to our problem

- EPS $\rightarrow$ already have a discretized $p$ (with $w_i = N^{-1}$)
- Tracking of $\phi$ and $\lambda$ is not practical because it forces to fully absorb uncertainty in $v$
- Not efficient
- Not operational
- Small feasible set (need to go slow to keep $v$ margin for unfavourable scenarios)
Flight plans

FL360 M0.81
FL380 M0.83
FL340 TAS240
**Dynamic system reformulation**

- Use ground distance \( r \) flown as independent variable. We can do this if \( v_G > 0 \) (true for realistic winds), as \( r(t) \) is strictly increasing.
- Groundspeed is \( v_G = \sqrt{(v \cos \psi + w_y)^2 + (v \sin \psi + w_x)^2} \).

**Reformulated cruise dynamical system**

\[
\frac{dx}{dr} = \frac{\dot{x}}{v_G} \\
\frac{dt}{dr} = v_G(r)^{-1}
\]

- Cost functional: \( J = \mathbb{E}[-m_f + \text{CI} \cdot t_f] + \text{DP}(t_f,\text{max} - t_f,\text{min}) \) with \( t_f,\text{min} \leq t_f,i \leq t_f,\text{max} \).
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Description

- A330 from NY to Lisbon
- Flying at M=.82 and FL380
- 20th of January, 2016
- 200 hPa level forecast from the PEARP ensemble (35 members, from Météo France)
Routes
Groundspeed

![Groundspeed graph](image)

Distance (km)

Ground speed (m/s)
Introduction

Methodology

Results

Discussion

Constant airspeed

Time lead or lag

![Graph showing time lead or lag vs distance (km)](image-url)
Time lead or lag

![Graph showing the relationship between distance and heading](image)

- **Course**
- **True heading**

Distance (km):
- 0
- 1000
- 2000
- 3000
- 4000
- 5000
- 6000

Heading (rad):
- 1.4
- 1.5
- 1.6
- 1.7
- 1.8
- 1.9
- 2.0

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Routes
Groundspeed
Time lead or lag

![Graph showing time lead or lag vs distance](image)
Time lead or lag

\[
\begin{align*}
\text{Distance (km)} & : 0, 1000, 2000, 3000, 4000, 5000, 6000, 7000 \\
\text{Heading (rad)} & : 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 \\
\text{Course} & \quad \text{True heading}
\end{align*}
\]
Predictability-efficiency trade-off

![Graph showing the relationship between fuel consumption and arrival time range with varying predictability (p).]
Routes (Variable velocity (CI=10))
Airspeeds (CI=10)
Predictability-efficiency trade-off
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Main takeaways

- We have developed a methodology to do flight planning under uncertainty and include predictability as an objective.
- Adapted to operational concepts.
- Scaling of $O(nN)$ is manageable (parallelization possibilities?)
- Same advantages and drawbacks of other direct methods (possibility of local optima but better handling of more complex, nonlinear and higher-dimensional systems compared to raw DP).
- Natural handling of EPS forecasts.
Future work

- Dynamic Weather, BADA 4
- Extension to 4D (altitude changes)
- Assessment of potential benefits
- Combination with other uncertainty sources
- Extension to tactical avoidance of thunderstorms
- Conflict avoidance & resolution?
Acknowledgements

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