Optimal Aircraft Path Planning in a Structured Airspace Using Ensemble Weather Forecasts

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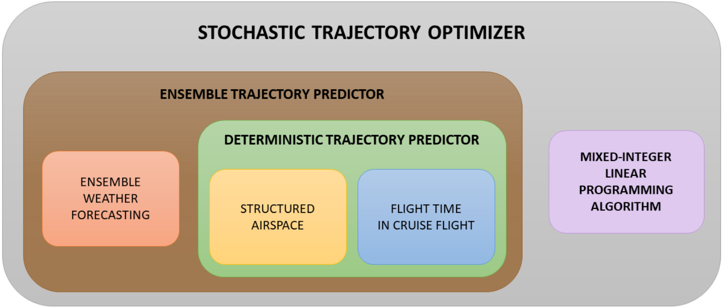
Introduction (I)

- **Trajectory optimization** is a subject of great importance in air traffic management (ATM), that aims at **defining optimal flight procedures** that leads to cost-efficient flights.
- A promising approach to improve the ATM system performance while maintaining high safety standards is **integrating uncertainty information**.
- Among the various uncertainty sources affecting the ATM, **weather uncertainty** has an **important impact** on the route planning process.

Introduction (II)

- **Main objective**:
  - To develop a stochastic methodology capable of finding the global-optimal aircraft path within a structured airspace taking into account uncertain winds and air temperature provided by an EPS.

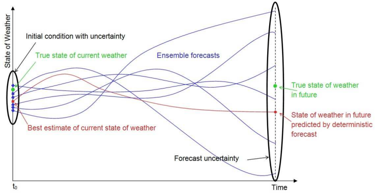


Methodology proposed for optimal path planning.

Ensemble Weather Forecasting

- Wind and temperature uncertainties are defined by an **Ensemble Prediction System** (EPS).

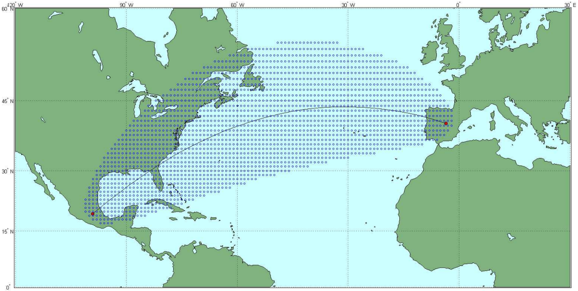
- An EPS is obtained by
  - slightly **altering the initial conditions** and/or physical parameters, and/or
  - considering time-lagged or **multi-model** approaches.

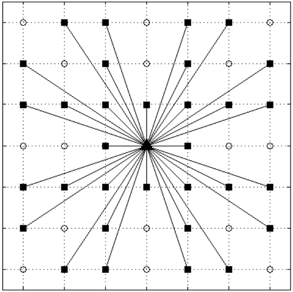


- An EPS constitutes a **representative sample** of the possible realizations of the **potential weather outcome**.

- The **uncertainty information is in the spread** of the various forecasts of the ensemble.

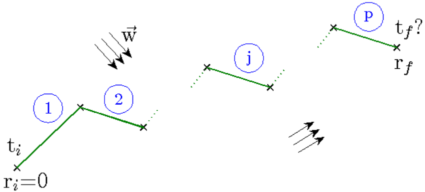
Airspace Structure: Search area and connections



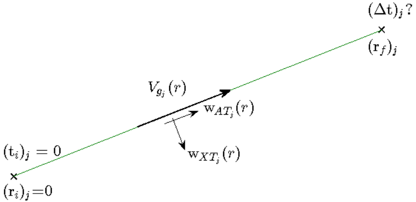


- Geographical search area:
    - **Trade-off** between **optimality** and **computational efficiency**.
    - **Uniform grid** restricted to be **inside a spherical ellipse** with foci located at arrival and destination airports.
- Each waypoint is connected to some of its neighbours in a 7x7-waypoint square centred at the waypoint of interest, what leads to a **great flexibility** to the aircraft optimal route.

Flight time in cruise flight (I)



- The cruise is **formed by p segments**, flown at:
  - **constant course**,
  - **constant M,** and
  - **constant pressure altitude**.



- Both **along-track winds** and **crosswinds** are considered.

$$V_{g_j}(r) = \sqrt{M^2 \kappa R_g T(r) - w_{XT_j}^2(r)} + w_{AT_j}(r)$$

Flight time in cruise flight (II)

- For a given distance flown from a waypoint (namely r)
  1. the **navigation equations** provide the actual coordinates $(\phi(r),\lambda(r))$,
  2. the air temperature and the wind vector are obtained from the EPS for that location; the latter is projected onto the constant-course segment, leading to $w_{AT_j}(r)$ and $w_{XT_j}(r)$.
- The **flight time** corresponding to **cruise segment** j is obtained from:

$$(\Delta t)_j = \int_0^{(r_f)_j} \frac{\mathrm{d}r}{V_{g_j}(r)}$$

- The **total flight time** is given by

$$t_f = \sum_{j=1}^{p} (\Delta t)_j$$

Ensemble trajectory prediction

- Ensemble trajectory prediction is one of the **main approaches commonly used** for trajectory prediction subject to uncertainty provided by the EPS.
- For each member k of the ensemble ($k = 1, ..., n$), a deterministic trajectory predictor is used, leading to an **ensemble of trajectories**.
- For a given cruise path compatible with the airspace structure, the final result is a set of n values for the cruise flight time (t_{f_1}, ...,t_{f_n}).
- Statistical characterization can be performed: **Mean and spread**.

Stochastic optimal path planning (I)

- **The aircraft route is a path in a graph**, as the cruise flight is composed of several segments connecting waypoints.
- Optimization of the aircraft route is, therefore, a **shortest path problem**.
- The graph is defined by a **set of nodes** (waypoints), a **set of links** (the constant-course segments) and a **set of link costs**.
- A **stochastic approach** has to be considered, with a model based on a **discrete set of scenarios** (the ensemble members).

Stochastic optimal path planning (II)

- The problem is stated as **finding the unique route** that minimizes a linear combination of the average total flight time (**best efficiency**) and a measurement of the spread of the trajectories (**best predictability**):

$$J = \frac{1}{n}\sum_{k=1}^{n} t_f^{[k]} + dp\left[\max_k\left(t_f^{[k]}\right) - \min_k\left(t_f^{[k]}\right)\right]$$

- The dispersion penalty parameter, *dp*, controls the **trade-off between efficiency and predictability**.

Stochastic optimal path planning (III)

- A combinatorial optimization approach is considered, with the following terms:

$$c^{[k]} \qquad x \qquad A_{eq} \qquad d \qquad \hat{c} = \frac{1}{n}\sum_{k=1}^{n} c^{[k]} \qquad l \qquad m$$

- **Mixed Integer Linear Programming Problem:**

$$\begin{cases} \min\limits_{x,y,z} & \hat{c}^T x + dp(y - z) \\ s.t. & A_{eq} x = d \\ & y \geq c^{[k]T} x \qquad \forall k \in \{1, .., n\} \\ & z \leq c^{[k]T} x \qquad \forall k \in \{1, .., n\} \\ & x = \{0, 1\}^l \\ & y \in \mathbb{R}, \; z \in \mathbb{R} \end{cases}$$

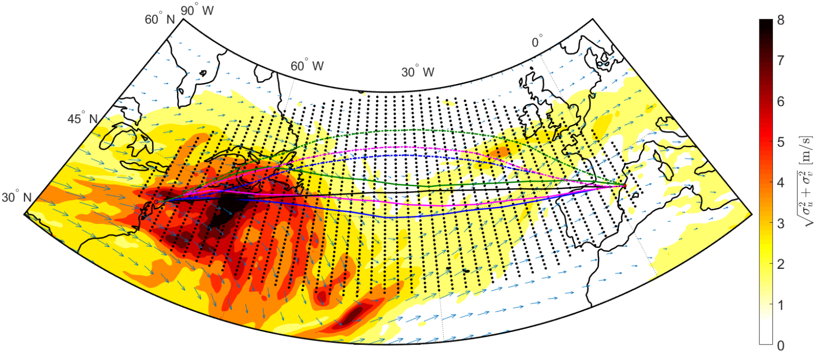
Application considered

- **EPS chosen:** ECMWF-EPS, released 1 March 2017 at 00:00, with a look-ahead time of 24 hours, and for pressure altitude 200 hPa (ISA altitude of 11784 m).

- **Flight considered:** From Philadelphia International Airport (KPHL) to Barcelona–El Prat Airport (LEBL) and viceversa, at Mach number $M = 0.82$, pressure altitude 200 hPa, and departing 1 March 2017 at 20:00.

	KPHL	**LEBL**
Latitude	39° 52' N	41° 18' N
Longitude	75° 15' W	2° 5' E

Optimal trajectories (I)

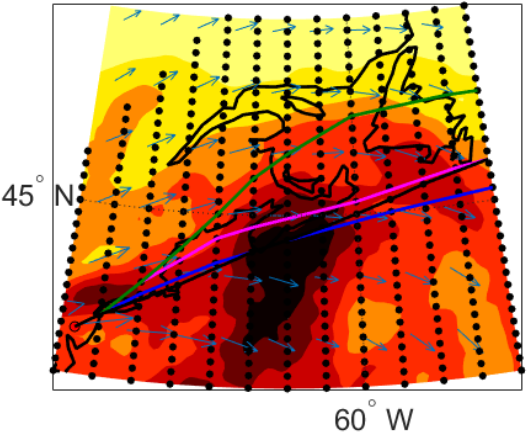


$dp = 0$ (blue solid), $E[t_f]$= 383.09 min, $\Delta[t_f]$= 5.60 min

$dp = 3$ (magenta solid), $E[t_f]$= 384.61 min, (+0.40%) $\Delta[t_f]$= 4.47 min (-20%)

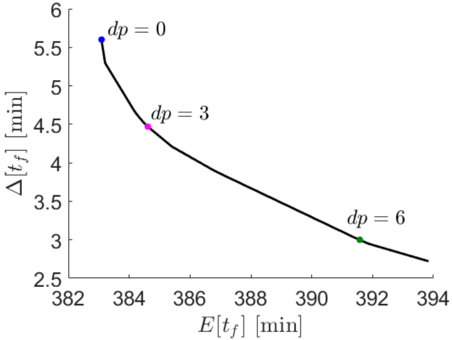
$dp = 6$ (green solid), $E[t_f]$= 391.58 min, (+2.2%) $\Delta[t_f]$= 3.00 min (-46%)

Optimal trajectories (II)



Pareto front

- The optimal pair of values ($E[t_f]$,$\Delta[t_f]$) obtained for different values of *dp* describe the Pareto frontier corresponding to the **conflicting objectives**, cost-efficiency and predictability.



- Main advantages of this curve:
  - it enables to present the **optimization results** to the Airspace Users (AUs) in a **clear and condensed form**, and
  - the AUs can make a **decision without revealing their cost structure**.

Final remarks

- A **stochastic methodology** has been implemented, which is capable of finding the **optimal aircraft path**, considering a **structured airspace**, in the presence of **uncertain winds and uncertain temperature** provided by an EPS.
  - Some **advantages** of applying this methodology have been **quantified**: A trade-off between efficiency and predictability can be performed.
- The consideration of **time-dependent wind and temperature fields** is left for future work.
  - Considerable **increase in complexity**: Mixed Integer Non-Linear Programming Problem.

Time-evolving EPS (I)

- The time to fly a segment depends on the time to reach the beginning of that segment.
- One possible approach: **Combinatorial optimization**
  - Extra vectors of decision variables $t^{[k]} \in \mathbb{R}^m$, one per ensemble member ($m \cdot n$ new unknowns).
  - Modified (non-linear) objective function:

$$J = \left[\frac{1}{n} \sum_{k=1}^{n} c^{[k]}(t^{[k]}) \right]^T x + dp(y - z)$$

  - New set of constraints stating the continuity of flight times along the optimal path ($ln + n$ new constraints, although very few are non trivial):

$$\begin{aligned} t_0^{[k]} &= t_{ini} \qquad \forall k \in \{1, .., n\} \\ \text{diag}\left[c^{[k]}(t^{[k]}) + A_{eq}^T t^{[k]}\right] x &= 0 \qquad \forall k \in \{1, .., n\} \end{aligned}$$

Time-evolving EPS (II)

- Another possible approach: **Evolutionary algorithms (direct search)**
  - For **any given path** x, a trajectory predictor is applied to obtain the associated cost, which takes into account an evolving meteorology.
  - Efforts are focused on the **generation of candidates** to optimal path, in such a way that these candidates converge to the global optimal solution.
  - Examples: Simulated annealing, particle swarm optimization, ant colonny, genetic algorithm, ...

Multiple weather hazards (I)

- Present in aircraft path planning: efficiency and safety criteria by condidering **wind fields** and **large-scale convection** areas.
- Other weather hazards such as **small scale convection** areas, **turbulence** and **icing** are tactically faced.
    - The associated **tactical interventions** are an important source of **delays**, **cost increments**, and **workload increments** (for pilots and air traffic controllers).
    - A more strategic view is needed, **facing the weather hazards more in advance** so as to reduce tactical interventions to exceptions.

Multiple weather hazards (II)

- Computation of the **convective risk** for each ensemble member:
  - form the information in the EPS, two convection indices are computed;
  - high risk regions are determined, in which both indices are above given thresholds;
  - the convective risk for any segment is given by the total distance flown inside high risk regions.
- Computations of the **turbulence risk** for each ensemble member is performed analogously, however based on different indices (turbulence metrics).
- **Avoiding convection and turbulence areas** is formulated as a **soft constraint**, that is, convective and turbulence costs are included in the objective function as a **linear combination with prescribed weights**.

Thank you

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