Optimal Aircraft Path Planning in a Structured Airspace Using Ensemble Weather Forecasts

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Trajectory optimization is a subject of great importance in air traffic management (ATM), that aims at defining optimal flight procedures that leads to cost-efficient flights.

A promising approach to improve the ATM system performance while maintaining high safety standards is integrating uncertainty information.

Among the various uncertainty sources affecting the ATM, weather uncertainty has an important impact on the route planning process.
Main objective:

To develop a stochastic methodology capable of finding the global-optimal aircraft path within a structured airspace taking into account uncertain winds and air temperature provided by an EPS.

Methodology proposed for optimal path planning.
Ensemble Weather Forecasting

- Wind and temperature uncertainties are defined by an Ensemble Prediction System (EPS).

- An EPS is obtained by
  - slightly altering the initial conditions and/or physical parameters, and/or
  - considering time-lagged or multi-model approaches.

- An EPS constitutes a representative sample of the possible realizations of the potential weather outcome.

- The uncertainty information is in the spread of the various forecasts of the ensemble.
Airspace Structure: Search area and connections

- Geographical search area:
  - **Trade-off** between **optimality** and **computational efficiency**.
  - **Uniform grid** restricted to be inside a **spherical ellipse** with foci located at arrival and destination airports.

- Each waypoint is connected to some of its neighbours in a 7x7-waypoint square centred at the waypoint of interest, what leads to a **great flexibility** to the aircraft optimal route.
Flight time in cruise flight (I)

- The cruise is **formed by** \( p \) **segments**, flown at:
  - constant course,
  - constant \( M \), and
  - constant pressure altitude.

- Both **along-track winds** and **crosswinds** are considered.

\[
V_{g_j}(r) = \sqrt{M^2 \kappa R_g T(r) - w_{XT_j}^2(r) + w_{AT_j}(r)}
\]
Flight time in cruise flight (II)

- For a given distance flown from a waypoint (namely \( r \))
  - the **navigation equations** provide the actual coordinates \((\phi(r), \lambda(r))\),
  - the air temperature and the wind vector are obtained from the EPS for that location; the latter is projected onto the constant-course segment, leading to \( w_{AT_j}(r) \) and \( w_{XT_j}(r) \).

- The **flight time** corresponding to cruise segment \( j \) is obtained from:
  \[
  (\Delta t)_j = \int_0^{(r_f)_j} \frac{dr}{V_{g_j}(r)}
  \]

- The **total flight time** is given by
  \[
  t_f = \sum_{j=1}^{p} (\Delta t)_j
  \]
Ensemble trajectory prediction

- Ensemble trajectory prediction is one of the **main approaches commonly used** for trajectory prediction subject to uncertainty provided by the EPS.

- For each member $k$ of the ensemble ($k = 1, \ldots, n$), a deterministic trajectory predictor is used, leading to an **ensemble of trajectories**.

- For a given cruise path compatible with the airspace structure, the final result is a set of $n$ values for the cruise flight time ($t_{f_1}, \ldots, t_{f_n}$).

- Statistical characterization can be performed: **Mean and spread**.
Stochastic optimal path planning (I)

- The aircraft route is a path in a graph, as the cruise flight is composed of several segments connecting waypoints.

- Optimization of the aircraft route is, therefore, a shortest path problem.

- The graph is defined by a set of nodes (waypoints), a set of links (the constant-course segments) and a set of link costs.

- A stochastic approach has to be considered, with a model based on a discrete set of scenarios (the ensemble members).
Stochastic optimal path planning (II)

The problem is stated as finding the unique route that minimizes a linear combination of the average total flight time (best efficiency) and a measurement of the spread of the trajectories (best predictability):

$$J = \frac{1}{n} \sum_{k=1}^{n} t_f^{[k]} + dp \left[ \max_k \left( t_f^{[k]} \right) - \min_k \left( t_f^{[k]} \right) \right]$$

The dispersion penalty parameter, $dp$, controls the trade-off between efficiency and predictability.
Stochastic optimal path planning (III)

- A combinatorial optimization approach is considered, with the following terms:

\[
\begin{align*}
    c^{[k]} x \quad A_{eq} d \quad \hat{c} = \frac{1}{n} \sum_{k=1}^{n} c^{[k]} \quad l \quad m
\end{align*}
\]

- **Mixed Integer Linear Programming Problem:**

\[
\begin{align*}
    \min_{x,y,z} \quad \hat{c}^T x + dp(y - z) \\
    \text{s.t.} \quad A_{eq} x = d \\
    y \geq c^{[k]T} x \quad \forall k \in \{1, \ldots, n\} \\
    z \leq c^{[k]T} x \quad \forall k \in \{1, \ldots, n\} \\
    x = \{0, 1\}^l \\
    y \in \mathbb{R}, \quad z \in \mathbb{R}
\end{align*}
\]
Application considered

- **EPS chosen**: ECMWF-EPS, released 1 March 2017 at 00:00, with a look-ahead time of 24 hours, and for pressure altitude 200 hPa (ISA altitude of 11784 m).

- **Flight considered**: From Philadelphia International Airport (KPHL) to Barcelona–El Prat Airport (LEBL) and vice versa, at Mach number $M = 0.82$, pressure altitude 200 hPa, and departing 1 March 2017 at 20:00.

<table>
<thead>
<tr>
<th>KPHL</th>
<th>LEBL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>39° 52’ N</td>
</tr>
<tr>
<td>Longitude</td>
<td>75° 15’ W</td>
</tr>
</tbody>
</table>
Optimal trajectories (I)

$dp = 0$ (blue solid), \hspace{1cm} $E[t_f] = 383.09$ min, \hspace{1cm} $\Delta[t_f] = 5.60$ min

$dp = 3$ (magenta solid), \hspace{1cm} $E[t_f] = 384.61$ min, (+0.40%) \hspace{1cm} $\Delta[t_f] = 4.47$ min (-20%)

$dp = 6$ (green solid), \hspace{1cm} $E[t_f] = 391.58$ min, (+2.2%) \hspace{1cm} $\Delta[t_f] = 3.00$ min (-46%)
Optimal trajectories (II)
The optimal pair of values \((E[t_f], \Delta[t_f])\) obtained for different values of \(dp\) describe the Pareto frontier corresponding to the conflicting objectives, cost-efficiency and predictability.

Main advantages of this curve:

- it enables to present the optimization results to the Airspace Users (AUs) in a clear and condensed form, and
- the AUs can make a decision without revealing their cost structure.
Final remarks

- A stochastic methodology has been implemented, which is capable of finding the optimal aircraft path, considering a structured airspace, in the presence of uncertain winds and uncertain temperature provided by an EPS.

  - Some advantages of applying this methodology have been quantified: A trade-off between efficiency and predictability can be performed.

- The consideration of time-dependent wind and temperature fields is left for future work.

  - Considerable increase in complexity: Mixed Integer Non-Linear Programming Problem.
Time-evolving EPS (I)

- The time to fly a segment depends on the time to reach the beginning of that segment.

- One possible approach: **Combinatorial optimization**
  - Extra vectors of decision variables $t^{[k]} \in \mathbb{R}^m$, one per ensemble member ($m \cdot n$ new unknowns).
  - Modified (non-linear) objective function:
    \[
    J = \left[ \frac{1}{n} \sum_{k=1}^{n} c^{[k]}(t^{[k]}) \right]^T x + dp(y - z)
    \]

- New set of constraints stating the continuity of flight times along the optimal path ($ln + n$ new constraints, although very few are non trivial):
  \[
  \text{diag} \left[ c^{[k]}(t^{[k]}) + A_{eq}^T t^{[k]} \right] x = 0 \quad \forall k \in \{1, .., n\}
  \]
  \[
  t_0^{[k]} = t_{ini} \quad \forall k \in \{1, .., n\}
  \]
Another possible approach: **Evolutionary algorithms (direct search)**

- For **any given path** \( x \), a trajectory predictor is applied to obtain the associated cost, which takes into account an evolving meteorology.

- Efforts are focused on the **generation of candidates** to optimal path, in such a way that these candidates converge to the global optimal solution.

- Examples: Simulated annealing, particle swarm optimization, ant colony, genetic algorithm, ...
Multiple weather hazards (I)

- Present in aircraft path planning: efficiency and safety criteria by considering wind fields and large-scale convection areas.

- Other weather hazards such as small scale convection areas, turbulence and icing are tactically faced.
  
  - The associated tactical interventions are an important source of delays, cost increments, and workload increments (for pilots and air traffic controllers).
  
  - A more strategic view is needed, facing the weather hazards more in advance so as to reduce tactical interventions to exceptions.
Multiple weather hazards (II)

- Computation of the **convective risk** for each ensemble member:
  - form the information in the EPS, two convection indices are computed;
  - high risk regions are determined, in which both indices are above given thresholds;
  - the convective risk for any segment is given by the total distance flown inside high risk regions.

- Computations of the **turbulence risk** for each ensemble member is performed analogously, however based on different indices (turbulence metrics).

- **Avoiding convection and turbulence areas** is formulated as a **soft constraint**, that is, convective and turbulence costs are included in the objective function as a **linear combination with prescribed weights**.
Thank you

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