Fuel Saving by Gradual Climb Procedure

Ryota Mori

(Electronic Navigation Research Institute)
Single Flight Optimal Trajectory

Optimal climb CAS: 320 kt

TOC (Top of climb)

Accelerate to 320 kt

Fuel optimal altitude increases with time
(The aircraft gets lighter with time)
Optimal climb CAS: 320 kt

Accelerate to 320 kt

T = T_{\text{max}}

Accelerate to 250 kt

250 kt maximum under 10000 ft or below

Noise abatement

TOC

Step climb

• Basically, the higher altitude is better in terms of fuel consumption.
Fuel Consumption and Thrust

• Maximum Continuous Thrust (MCT) / Maximum Climb Thrust (MCL)… so-called maximum thrust
  – MCT is not the most fuel efficient, because the engine is designed to be the most fuel efficient during cruise.
  – Saving in climb thrust is achieved by reducing the rate of climb (ROC).

• The aircraft is more fuel efficient at higher altitude.
  – Small ROC means TOC moves further.
  → Which impact is bigger?
    1) “Fuel saving by lower thrust” or
    2) “Fuel saving to reach TOC earlier with MCT”? 
  → Numerical optimization approach
Problem formulation (1)

Aircraft dynamics (no wind)
\[
\begin{align*}
\dot{x} &= v \cos \gamma \\
\dot{z} &= v \sin \gamma \\
\dot{v} &= \frac{T - D}{m} - g \sin \gamma \\
\dot{\gamma} &= \frac{L}{mv} - \frac{g \cos \gamma}{v} \\
\dot{m} &= f(M, T) \\
T &= T_{\text{ratio}} (T_{\text{max}} - T_{\text{min}}) + T_{\text{min}} \\
T_{\text{ratio}} &= [0 \quad 1]
\end{align*}
\]

Control (optimization) variables
\[
u = [\dot{\gamma}, T_{\text{ratio}}]^T
\]

- **Point mass model**
  - No wind considered
  - No lateral motion considered
- **BADA 4 model**
  - B777-300 (Engine: GE)
- **Objective function to be minimized:**
  \[
  J = \frac{100}{3600} CI \cdot t_f + 0.453592 \int_0^{t_f} -\dot{m} dt
  \]
  - Flight time
  - Fuel consumption
  - Cost index:
    - 100 [100 lb/hour]
    (= 2.78 lb / second)
  - Unit: [lb]
Problem formulation (2)

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial altitude</td>
<td>30000 ft</td>
</tr>
<tr>
<td>Terminal altitude</td>
<td>36000 ft</td>
</tr>
<tr>
<td>Initial/Terminal Mach</td>
<td>0.83</td>
</tr>
<tr>
<td>Initial climb angle</td>
<td>1.0 deg</td>
</tr>
<tr>
<td>Flight distance</td>
<td>500 NM</td>
</tr>
<tr>
<td>Initial Weight</td>
<td>540,000 lb</td>
</tr>
</tbody>
</table>

- Optimal climb: single-stage optimal control problem
- MCT climb: 2-stage optimal control problem
  ➔ NLP (Nonlinear programming) solver is used.
Optimal Trajectory and Current MCT Trajectory

Optimal climb profile cannot be implemented in the current FMC and ATC. ➔ Sub-optimal practical climb profile is proposed.
Purpose of This Research

• To propose a new climb profile which saves fuel compared to the MCT climb.
  – The new profile should be possible within the current FMC.
  – Additional pilot tasks should be minimized.
  – Negative impact to ATC should be minimized.
  – Potential fuel savings should not be negligible.
  • The cumulative effect is also important because most aircraft are expected to change the climb profile.
Current FMC Climb

• Two basic FMC modes to climb
  – VNAV SPD is usually applied during climb.
  – V/S (vertical speed) mode requires a target vertical speed
    • Climb trajectory can be changed.
      ➔ V/S mode is used here.

<table>
<thead>
<tr>
<th></th>
<th>VNAV SPD mode</th>
<th>V/S mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch control</td>
<td>Track target speed</td>
<td>Track target V/S</td>
</tr>
<tr>
<td>Thrust control</td>
<td>MCT</td>
<td>Track target speed</td>
</tr>
<tr>
<td>Target speed</td>
<td>Calculated by FMC</td>
<td>Calculated by FMC</td>
</tr>
<tr>
<td>Note</td>
<td>V/S is uniquely determined from the thrust.</td>
<td>Target V/S is set manually.</td>
</tr>
</tbody>
</table>
Proposed Climb Profile

- Climb with MCT to “transfer altitude”.
- Climb with constant V/S to cruise altitude.
Calculation Conditions

• B777-300
  – About 2000 ft/min ROC with MCT
• Two scenarios
  – Climb to optimal altitude (36000 ft @ 540000 lb)
  – Climb to lower altitude (34000 ft) due to ATC instruction
• Target V/S
  – 500 ft/min or 1000 ft/min
• Transfer altitude
  – 30000 ft - 34000 ft
• 3-stage optimization
  – MCT climb
  – Constant V/S climb
  – Cruise

Optimal profiles under two scenarios
Fuel Saving by Proposed Procedure

- Proposed procedure saves 30-40 lb fuel if appropriate transfer altitude and V/S are chosen.
  - TOC moves 1-6 min/10-50 NM forward.
Impacts to Pilots & ATC

- Impacts to pilots
  - Some additional tasks are needed.
  - Minimum 500 ft/min ROC is recommended for situational awareness in TCAS monitor.
- Impacts to ATC
  - No negative effect will be observed unless there is other traffic.
    - ATC does not instruct V/S or time limit of climb.
  - 1-6 minutes delay to reach TOC might be an issue when there is other traffic nearby.
  - 500 ft/min climb is not slow.
    - V/S with MCT near TOC is less than 1000 ft/min for most aircraft types.

**VNAV PATH**
- Push VNAV button.

**VNAV SPD**
- Push V/S button.
- Select target V/S.
• During step climb, MCT is usually applied.
  – Low ROC with V/S mode will save fuel.
• 3-stage optimization (cruise-step-cruise)
  – Appropriate constraints are set in “step” stage.
## Calculation Results for Step Climb

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft type</td>
<td>B777-300</td>
<td>B777-300</td>
<td>A330-300</td>
</tr>
<tr>
<td>Initial weight [lb]</td>
<td>540,000</td>
<td>610,000</td>
<td>440,000</td>
</tr>
<tr>
<td>Initial altitude [ft]</td>
<td>36,000</td>
<td>33,000</td>
<td>37,000</td>
</tr>
<tr>
<td>Terminal altitude [ft]</td>
<td>38,000</td>
<td>35,000</td>
<td>39,000</td>
</tr>
<tr>
<td>Initial/Terminal Mach</td>
<td>0.83</td>
<td>0.83</td>
<td>0.80</td>
</tr>
<tr>
<td>Cost index</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Flight distance [NM]</td>
<td>2,000</td>
<td>2,000</td>
<td>2,000</td>
</tr>
<tr>
<td><strong>Objective function [lb] (Compared to MCT climb)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal climb</td>
<td>−88</td>
<td>−93</td>
<td>−71</td>
</tr>
<tr>
<td>50 ft/min climb</td>
<td>−55</td>
<td>−57</td>
<td>−41</td>
</tr>
<tr>
<td><strong>500 ft/min climb</strong></td>
<td>−28</td>
<td>−29</td>
<td>−14</td>
</tr>
<tr>
<td>1000 ft/min climb</td>
<td>−14</td>
<td>−17</td>
<td>−1</td>
</tr>
<tr>
<td>MCT climb</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Conclusions

- A new practical climb procedure (gradual climb procedure) is proposed.
  - Gradual climb procedure is applicable in the current FMC using V/S mode.
  - 30-40 lb fuel saving per climb is expected with B777-300.
    - Cumulative effect will be significant because most departure aircraft can apply this procedure.
  - Negative effects to pilots and ATC are limited.
    - Pilots have to perform some additional tasks.
  - The similar procedure can be applied in step climb procedure.
    - Step climb is operated by long-haul flights only.
- Detailed conditions (appropriate V/S & transfer altitude with temperature, wind, aircraft type, etc) will be further investigated.
Fuel Saving by Gradual Climb Procedure

Thank you for your attention!

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### Scenario 1
*(Climb to 36000ft)*

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>MCT</td>
<td>-</td>
<td>27160</td>
<td>16700</td>
<td>3765</td>
</tr>
<tr>
<td>Optimal</td>
<td>-</td>
<td>27095</td>
<td>16653</td>
<td>3759</td>
</tr>
<tr>
<td>500</td>
<td>33000</td>
<td>27119</td>
<td>16663</td>
<td>3764</td>
</tr>
<tr>
<td>1000</td>
<td>30000</td>
<td>27118</td>
<td>16666</td>
<td>3763</td>
</tr>
</tbody>
</table>

![Graph of altitude vs. flight distance](image)