Continuous Climb Operations
with Minimum Fuel Burn

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Abstract—Continuous climb operations are one of several instruments, developed by the Single European Sky ATM Research program SESAR to improve the environmental compatibility of the future air traffic management by designing an air traffic system with minimum environmental impact and minimum direct operational costs at an increased safety level, compared to today. With respect to minimum fuel flow, the optimum continuous climb is defined as a continuously climb profile with a minimum number of level-offs and thrust changes to avoid superfluous acceleration forces. This definition results in highly aircraft specific and weather dependent continuous climb profiles which are hardly to predict. In this paper, the optimum climb profiles of four different aircraft types are estimated under real atmospheric conditions by modeling trajectories with an aircraft performance model, which is specialized to unsteady flows. We found, that even in realistic weather conditions, the target function of the aspired true air speed is very similar to the objective of climbing with a maximum climb rate, which corresponds to a minimum fuel climb profile in a standard atmosphere. The cruising altitude and a corresponding true air speed with respect to a maximum specific range is more important for an optimized continuous climb, than the climb gradient.

I. INTRODUCTION

The growing public awareness of the aviation impact on climate change forces aviation stakeholders more and more to search for climate friendly solutions. Optimization potential has been found on almost every air traffic planning level ranging from network design to fleet assignment and trajectory optimization [1]. Regardless of the planning level, flight performance modeling is necessary for a reliable optimization of the air traffic system, because it is the smallest unit, on which each air traffic optimization should be based. Since the foundation of the Single European Sky (SES) and the corresponding research program Single European Sky ATM Research (SESAR) in 1999, the reduction of the air traffic environmental impact is regulated by law [2] and the targets pose a challenge for all air traffic stakeholders. Beside the triplication of capacity, the increase of safety by a factor of 10 and the decrease of air traffic management costs by 50 %, the environmental compatibility of each flight should be reduced by 10 % [2]. With the SESAR Master plan [1], the basic concept for the design of the future air traffic management (ATM) had been established by EUROCONTROL in 2012 to meet the SESAR targets by the introduction of an optimized flight trajectory, amongst others [1]. Therewith, the SESAR targets shall be carried out by ”Moving from Airspace to 4D Trajectory Management”. The first step towards this action is called the “Time-based operations” and focusses on the deployment of airborne trajectories [1], which considers all constraints inflicted by the highly complex and dynamic environmental conditions [3]. Therewith, the large and important impact of atmospheric conditions on trajectory optimization has been identified. Free routing is aspired, to enable optimized trajectories [1] under real weather conditions. Free routes are freely planed routes between a defined entry point and a defined exit point constraint by published or unpublished waypoints [1].

Therewith, the ground for innovative concepts in the trajectory optimization is prepared and several ideas have been formulated in the SESAR Master plan [1] and in the SESAR Concept Of Operations (CONOPS) Step 1 [4]. Amongst others, continuous climb and descent operations have been identified as promising concepts for an environmentally friendly trajectory optimization. While continuous descent operations (CDO) are already established and evaluated in flight operations [5], continuous climb operations (CCO) still need to be implemented by Air Navigation Service Providers (ANSPs). CCOs are presumed as fuel efficient, noise reducing and by a reduced amount of workload for the flight crew and the controller [6]. These benefits, compared to conventional climb profiles, are established by the renunciation of level-off segments, whereby noisy and inefficient acceleration phases are avoided and intermediate altitude clearances during climb are no longer applied [7]. Following the Continuous Climb Operation (CCO) Manual, composed by the International Civil Aviation Organization (ICAO) as Doc 9993 AN/495 [6], CCOs describe uninterrupted departures and climb flights in a maximum possible magnitude, without any level-offs up to cruising altitude [6]. This climb should be operated with optimum engine thrust and climb speeds [6]. This description contains more general statements, than precise recommendations for the operational implementation in the flight planning [8] [9] [10]. Lots of uncertainties regarding the optimum climb angle γ, thrust setting $F_T$ and speed $v_{TAS}$, especially under realistic weather conditions, could not be cleared by field studies to this day [11] [12]. Additionally, concerning the Air Traffic Control (ATC), conflicts between arriving and departing air traffic can not be excluded by now.
The precise qualitative estimation of the parameters for the design of an optimum CCO depends on several variables, which are functions of both aircraft type and atmospheric conditions during the whole flight. Therefore, an aircraft type specific performance model is necessary, respecting at least 3D atmospheric weather information and assuring only physically possible flights by considering unsteady flows at each time step. For this purpose, the Compromised Aircraft performance model with Limited Accuracy (COALA) has been developed [13], which derives the target figure (i.e. the true airspeed $v_{\text{TAS}}$) from target functions, controls $v_{\text{TAS}}$ with a proportional-integral-derivative (PID) controller and achieves the 4D trajectory by the integration of the dynamic equation. With COALA, optimum climb angles, target functions of $v_{\text{TAS}}$ and thrust settings have been identified and discussed for four aircraft types (an European common used narrow-body medium range aircraft $A$, an American common used narrow-body medium range aircraft $B$, an American long-range wide-body twin-engine freighter $C$ and an American long-range wide-body three-engine freighter $M$) using 3D Grib2 (GRIDded Binary) weather data provided by the National Weather Service NOAA [14].

II. FLIGHT PERFORMANCE MODELING OF CONTINUOUS CLIMB OPERATIONS

For the precise estimation of the CCO, the whole trajectory has to be modeled, because the CCO profile depends on the requirements of the aspired cruise flight at the top of climb (TOC) on the aircraft performance, i.e. on $v_{\text{TAS}}$, and cruising pressure $p_{\text{cruise}}$.

The 4D flight performance model COALA combines the impact of aircraft specific aerodynamics and the important influence of 3D weather information both affecting an optimized trajectory in the actual operational flight. COALA calculates $v_{\text{TAS}}$, thrust $F_T$, fuel flow $m_f$, forces of acceleration $a_x$ and $a_y$, flight path angle $\gamma$, time of flight $t$, and the emission quantities of $CO_2$, $H_2O$, $NO_x$, $BC$, $CO$, $HC$, $SO_2$ and $H_2SO_4$ and considers aircraft type specific aerodynamical parameters like wing area $S$, maximum Mach number $MMO$, number of engines, aircraft weights and the drag polar depending on flap hand position and Mach number [13].

For multi-objective optimization, COALA uses target functions for $v_{\text{TAS}}$ and flight path angle $\gamma$, which are continuously calculated for each time step and used as controlled variable and are controlled with the lift coefficient as regulating variable. Aerodynamical and flight performance specific limits are considered all the time. Under real weather conditions the aircraft performance modeling turns into an unsteady system, because speed and acceleration are subject to constant changes, which are not negligible. Therewith, forces of acceleration are considered and minimized all the time and only physically possible trajectories are calculated. The dynamic equation considering the horizontal and vertical plane is integrated for the estimation of the achieved $v_{\text{TAS}}$ and air distance. Ground speed $v_{\text{GFS}}$ and ground distance are estimated considering wind speeds for Eastward $u$ and Northward $v$ directions [13].

Due to missing calibration data, some aircraft specific data have to be taken from the Base of Aircraft DAta (BADA), provided by EUROCONTROL [15] [16]. BADA data are used for the approximation of the drag polar, i.e. the functional relationship between drag coefficient $c_D$ and lift coefficient $c_A$, the maximum climb thrust $MCL$ and the fuel flow $m_f$. Therefore, a limited accuracy has to be accepted. Regarding $m_f$, errors of “less than 5%” for BADA 3 and “well below 5%” for BADA 4 are considered [17]. If the aircraft type is available, BADA 4.1 will be used.

For optimization, target functions for the ISA standard atmosphere are taken from Scheiderer [18] and Kaiser [19] and are applied to atmospheric conditions, which are not analytically describable, by numerical extremum estimations of the target functions [13]. By default, COALA optimizes the trajectory with respect to minimum fuel burn (i.e. cost index $CI = 0$). This target is achieved by aspired true air speeds for a maximum climb angle $\gamma$ [rad] up to the safety level of 10000 ft, a maximum climb rate $w$ [$m/s^1$]

\[
w = \sin \gamma \cdot v_{\text{TAS}}
\]  

above 10000 ft during continuous climb, a maximum specific Range $R$ [$m/kg^{-1}$]

\[
R = \frac{v_{\text{TAS}}}{m_f}
\]  
during cruise and a maximum lift/drag ratio $E$ [a.u.]

\[
E = \frac{F_L}{F_D}
\]
during continuous descent. In Equation 3, $F_L$ and $F_D$ denote the lift force [N] and the drag force [N], respectively. The target functions for $v_{\text{TAS}}$ are variable and can be manipulated by a relative adjustment factor $\alpha$ of $v_{\text{TAS}}$, causing a steeper climb profile in the case of a deceleration of $v_{\text{TAS}}$ (i.e. $\alpha < 1$) with a lower true air speed at the top of climb TOC. A higher $v_{\text{TAS}}$ ($\alpha > 1$) causes a shallower climb profile with a higher true air speed at TOC. Fig. 1 shows the impact of this adjustment on the climb profile of aircraft $A$ with otherwise identical parameters. For $\alpha = 0.9$ a boundary of the aircraft flight performance is reached at cruising altitude $h_{\text{cruise}} = 220$ hPa, the true air speed at TOC is too low. Once at cruising altitude, the aircraft can not keep the aspired altitude and has to accelerate in the direction of the flight path, which leads to a temporarily decreased lift coefficient $c_A$ and a slightly loss of height.

III. AIRCRAFT SPECIFIC CONTINUOUS CLIMB OPTIMIZATION

The conditions of CCO for an optimized trajectory with minimum fuel burn are estimated for four different aircraft types under identical weather conditions on 4th of September 2016, 12 p.m.. The air connection between Frankfurt (EDDF) and Dubai (OMDB) is simulated. The single aisle aircraft (aircraft $A$ and aircraft $B$) are loaded with 10000 kg payload.
and 17000 kg fuel. The freighter aircraft (aircraft $M$ and aircraft $C$) have 70000 kg payload and 70000 kg fuel. Table I contains the main aircraft characteristics with major influence on the optimum continuous flight profile.

For take-off, thrust is linearly approximated as function of $v_{\text{TAS}}$. To optimize the continuous climb conditions, the adjustment factor $\alpha$ of the target function of $v_{\text{TAS}}$ during the second climb phase and the cruising pressure altitude $p_{\text{cruise}}$ act

and $\alpha = 0.9, 1.0, 1.1$ on the climb profile of aircraft $A$ climbing up to $p_{\text{cruise}} = 220$ hPa.

**Figure 1.** Influence of the speed adjustment $\alpha$ on $v_{\text{TAS}}$ above 10000 ft ($\approx 700$ hPa) on the climb profile of aircraft $A$ climbing up to $p_{\text{cruise}} = 220$ hPa.

**Figure 2.** Modeled trajectories of aircraft $A$ (blue) and aircraft $B$ (orange) from EDDF to OMDB for constant given input parameters $p_{\text{cruise}} = 220$ hPa and $\alpha = 1.0$.

**Figure 3.** Modeled true air speed of aircraft $A$ (blue) and aircraft $B$ (orange) during cruise.

**Figure 4.** Simulated trajectories of aircraft $M$ (blue) and aircraft $C$ (orange) from EDDF to OMDB with identical input parameters $p_{\text{cruise}} = 220$ hPa and $\alpha = 1.0$.

To optimize the continuous climb conditions, the adjustment factor $\alpha$ of the target function of $v_{\text{TAS}}$ during the second climb phase and the cruising pressure altitude $p_{\text{cruise}}$ act

**TABLE I**

| Characteristics of the modeled aircraft with main impact on flight performance. The following abbreviations are used: operating empty weight OEW, take-off weight TOW, maximum operating Mach number MMO, aircraft wing area $S$, total engine rated output $F_{00}$ |
|---------------------------------------------------------------|---------------------------------------------------------------|
| $OEW$ [t]                      | 39.0              | 41.4              | 145.1              | 127.4              |
| $TOW$ [t]                      | 66.0              | 68.4              | 285.1              | 267.4              |
| $MMO$ [au]                     | 0.82              | 0.82              | 0.87               | 0.87               |
| $S$ [m$^2$]                    | 122.6             | 124.5             | 427.8              | 338.9              |
| $F_{00}$ [kN]                  | 235.6             | 236.0             | 809.8              | 985.2              |

Drag polar causes a steeper climb profile of the aircraft $C$ with higher true air speeds (compare Fig. 5) and a shallower descent angle. Due to a higher $OEW$ and a lower drag polar, lower true air speeds are aspired for the aircraft $C$ during cruise.
as free variables and are varied $0.9 \leq \alpha \leq 1.12$ and
$260 \leq p_{\text{cruise}} \leq 190$ hPa, respectively. Therewith, the climb angle $\gamma$, the energy share between potential and kinetic energy, the true air speed along the climb vector, the cruising pressure altitude and the cruising true air speed are influenced and an optimum trajectory is identified. Following the criteria for an optimum continuous climb given by [6], changes in thrust and speed should be avoided. Hence, the aircraft has to reach the cruising altitude with a true air speed similar to the aspired cruising air speed. The aspired cruising air speed for minimum fuel burn $v_{\text{TAS}, R}$ is the true air speed for a maximum specific range $R_{\text{max}}$ (compare Equation 2) and depends on altitude, because fuel flow depends on altitude and on flight performance (i.e. available thrust, fuel flow, wing area, aircraft mass and drag polar). In turn, the optimum cruising altitude for minimum fuel burn is reached at the crossover altitude, where the high speed buffet [21]

$$v_{\text{TAS}, \text{HSB}} = M M O \sqrt{\kappa R_s T}$$

(4)

is equal to the true air speed $v_{\text{TAS}, R}$ at $R_{\text{max}}$ [19]. In Equation 4, $\kappa = 1.4$ [a.u.] denotes the adiabatic index, $R_s = 287.15$ (kg K)$^{-1}$ the individual gas constant of dry air and $T$ [K] the ambient air temperature. $v_{\text{TAS}, R}$ is a function of available thrust, fuel flow, wing area, aircraft mass and drag polar. Hence, both measures ($v_{\text{TAS}, R}$ and optimum pressure altitude $p_{\text{cruise}}$) are aircraft specific and weather specific, they even change along the flight path and cannot be predicted. From this follows, that optimum pressure altitude $p_{\text{cruise}}$ and optimum speed adjustment factor $\alpha$ have to be identified iteratively. Furthermore, these variables are not constant for different aircraft types or different atmospheric conditions. With the variation of $p_{\text{cruise}}$, the optimum cruising altitude with minimum fuel burn is identified. With the change in $\alpha$, the optimum true air speed at TOC is estimated.

The impact of $p_{\text{cruise}}$ and $\alpha$ on fuel burn is exemplified in Fig. 6 and Fig. 7 for aircraft $A$ and aircraft $C$. Here, fuel burn is calculated for several combinations of the varied input parameters $p_{\text{cruise}}$ and $\alpha$. The x-axis is the cruising pressure $p_{\text{cruise}}$ and each colored isoline shows a given value of $\alpha$. For the aircraft $A$, the expected functional relationship between fuel burn and cruising pressure is visible. Except for $\alpha = 0.9$, the influence of $\alpha$ is small, compared to the impact of $p_{\text{cruise}}$. $\alpha = 1.09$ shows the boundary of the aircraft performance: An increased true airspeed by 9% is hardly achievable and the fuel flow strongly increases (compare Fig. 6). The higher the cruising altitude, the more significant is the impact of $\alpha$.

On the other hand, the aircraft $C$ can deal with higher variations of the true air speed (Fig. 7), but is not able to climb up to higher cruising altitudes than $p_{\text{cruise}} = 220$ hPa at the given TOW and along the relatively short distance of $\approx 2700$ NM. At $p_{\text{cruise}} = 250$ hPa is a local minimum, which must be aircraft specific, because all aircraft are using the same lateral path at each flight level. Maybe, at $p = 220$ hPa the transition into cruise flight is easier for the steep climbing aircraft $C$ than for a shallow climbing aircraft $A$. The transition is at a different geographical location for the aircraft $C$, than for the aircraft $A$, with different weather conditions. Overall, the service ceiling $p_{\text{cruise}} = 220$ hPa of the aircraft $C$ seems to be the most fuel efficient cruising altitude. For all aircraft can be concluded, that cruising pressure $p_{\text{cruise}}$ has a larger influence on the fuel efficiency of the continuous climb profile, than the speed adjustment. Although the simulations are done in a real atmospheric environment, the trajectory with minimum fuel burn is identified in the very vicinity of $\alpha = 0.9$, i.e. for a climbing profile with maximum climb rate $w$. This has been already proposed by Scheiderer [18] and Kaiser [19] for the ISA atmosphere without wind consideration. From this follows, at the TOC and under the given atmospheric conditions, the climb speed is similar to the cruising speed and no acceleration forces are necessary at the TOC.

The results of the optimization of both parameters and the impact of this optimization on fuel burn can be taken from Tab. II. Again, it gets clear, that $\alpha = 1$ is a good choice for a fuel minimum continuous climb profile. Differences in fuel burn between $\alpha = 1$ and the optimized $\alpha$ at the optimized cruising pressure altitude are small (a few kilograms) and beyond the accuracy of the approximated fuel flow by BADA. The aircraft $M$ should climb with a reduced true air speed by 2%, because the target functions of $v_{\text{TAS}}$ are very high.
which often exceed the high speed buffet $v_{\text{TAS,HSB}}$ and must be bounded [13].

Finally, the optimized trajectories are compared in Fig. 8 and Fig. 9. Slightly different cruising altitudes and strongly different climb profiles, which do not originate from strongly different target functions for the true air speed during climb are identified. Different climb profiles between aircraft $M$ and aircraft $C$ mostly occur due to a lower drag polar of the aircraft $M$.

### IV. Conclusion

In this paper, optimum climb profiles with respect to minimum fuel burn of four different aircraft types are identified under real weather conditions. Therefore, trajectory optimization with the aircraft performance model COALA, which generates only physically possible trajectories and considers unsteady flows at each time step has been used. The target function of the true air speed has been varied to simulate different climb angles and climb speeds. Furthermore the optimum cruising altitude under the given conditions has been identified for each aircraft type. We found, that climbing with a maximum climb rate $w$ up to an environmentally influenced and aircraft specific cruising altitude leads to nearly fuel minimum trajectories. Furthermore, the aspired cruising pressure altitude strongly influences the optimum climb angle during continuous climb. The proposed criteria of an optimum CCO by [6] fully describe the procedure and are purposeful for trajectory optimization. However, the modeled results are not generalize able due to a strong influence of wind speed and wind direction and due to aircraft specific flight performance with a large impact on the trajectory, even under constant target functions. Furthermore, the resultant differences in flight performance have to be considered carefully and critically, due different approximations of fuel flow and drag polar between BADA 3 and BADA 4. Additionally, the errors in fuel flow, done by the BADA approximation ($\Delta m_f < 5\%$) always have to be in mind.

More work has to be done regarding the validation of the flight performance model, which is difficult, because waypoint based trajectories can not be flown and therewith, the comparison of simulated flights with Flight Operation Data (FODA) sets, provided by real aircraft during flight is difficult.

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