Discussion on Complexity and TCAS indicators for Coherent Safety Net Transitions

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SESAR Innovation Days - 08th November 2016

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Introduction

Objectives

AGENT

Methodology

- TCAS Indicators
- Complexity indicator
- Encounter Model

Results

- Range
- Sensitivity with encounter headings

Conclusions and further work

Objectives and Methodology

Objectives

- Introduction of one of the proposed Complexity indicators within AGENT
- Comparison of the TCAS Horizontal Detection indicators (tau and tau_mod) with the Complexity at the transition zone between Separation Management and Collision Avoidance

Methodology

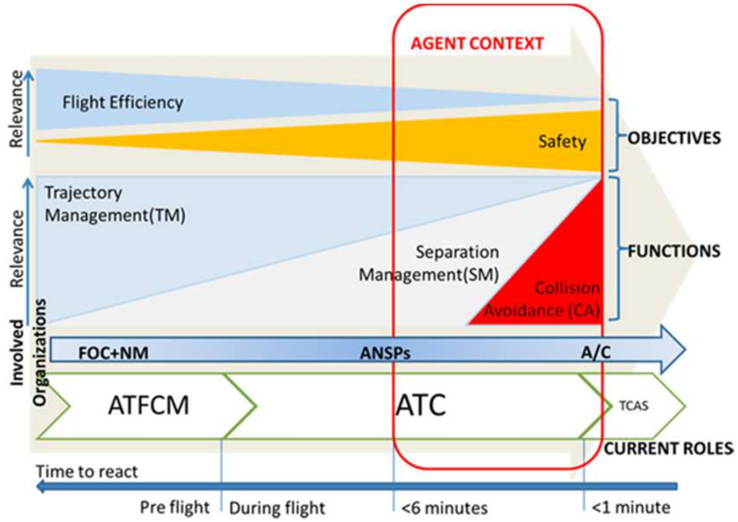
- Definition of an encounter model
- Parameters selection for the complexity indicator
- Comparison of the TCAS Horizontal Detection indicators (tau and tau_mod) with the complexity for CA detection:
  - Range when the alert is triggered;
  - Sensitivity to relative speeds and encounter angle.

Project Introduction

AGENT Adaptive self-Governed aerial Ecosystem by Negotiated Traffic

AGENT Mission: To perform a flight efficient, safe collaborative and supervised separation management, operationally integrated to trajectory management and collision avoidance.

- Multi-aircraft CDR
- Negotiated resolutions



Consortium

UAB Universitat Autònoma de Barcelona · ASLOGIC · Technische Universität Braunschweig · Cranfield University

Sponsors

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Complexity Metric I

"Density" notion

$$\frac{R^2 H}{8} \sum_{n=1}^{N} \frac{1}{\max(d_{mn}, R/2)^2 \cdot \max(h_{mn}, H/2)}$$

d_{mn}: horizontal range between Aircraft *m* and Aircraft *n*

T_{SM}: Characteristic Separation Management Time

"Intrinsic Complexity" notion

- Based on [D. Delahaye and S. Puechmorel, "Air traffic complexity based on dynamical systems," *Proc. IEEE Conf. Decis. Control*, pp. 2069–2074, 2010]

$$\dot{\boldsymbol{X}} = f(\boldsymbol{X}) \longrightarrow \dot{\boldsymbol{X}} - \dot{\boldsymbol{X}}_{\boldsymbol{m}} \approx \boldsymbol{J}_{\boldsymbol{f}} \cdot (\boldsymbol{X} - \boldsymbol{X}_{\boldsymbol{m}}) \longrightarrow \sum_{k=1}^{3} e^{-T_{SM} \cdot real(eig(k))}$$

$$CM_m = \left[\frac{R^2 H}{8} \sum_{n=1}^{N} \frac{1}{\max(d_{mn}, R/2)^2 \cdot \max(h_{mn}, H/2)}\right] \cdot \left[\sum_{k=1}^{3} e^{-T_{SM} \cdot real(eig(k))}\right]$$

Complexity Metric II

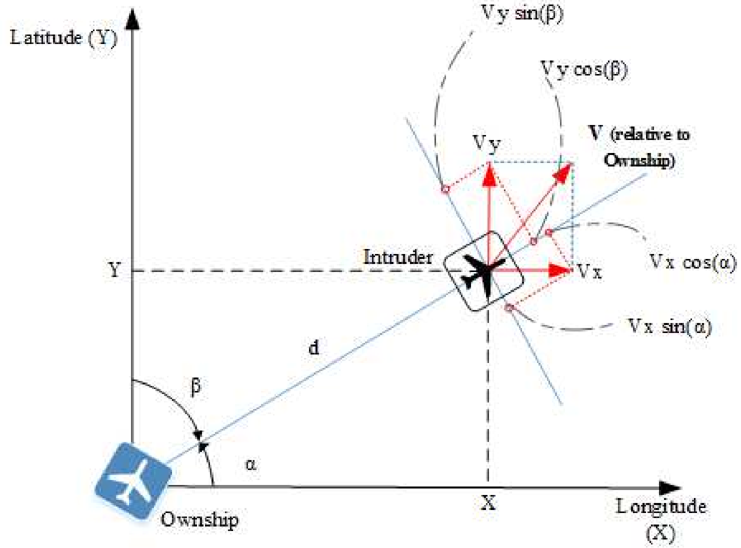
Pair-wise encounter

$$\dot{d} = V_x \cdot \cos(\alpha) + V_y \cdot \cos(\beta) + V_z \cdot \cos(\gamma)$$

Projecting to the reference axes

$$[\dot{\boldsymbol{X}} - \dot{\boldsymbol{X}}_{\boldsymbol{m}}] \cong \frac{V_x \cdot \cos(\alpha) + V_y \cdot \cos(\beta) + V_z \cdot \cos(\gamma)}{d} \cdot \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \cdot [\boldsymbol{X} - \boldsymbol{X}_{\boldsymbol{m}}]$$

Splitting on horizontal and vertical components, and considering the scalar form of the closure rate



$$\dot{d}_H = \frac{\boldsymbol{V} \cdot \boldsymbol{X_H}}{(d_H)^2} d_H \;\longrightarrow\; \lambda_H = \frac{\boldsymbol{V_H} \cdot \boldsymbol{X_H}}{{d_H}^2} \;\longrightarrow\; CM_H \cong \left[1 + \frac{R^2}{4} \cdot \frac{1}{(\max(\frac{R}{2}, \mathrm{r}))^2}\right] \left[e^{-T_{SM} \cdot \lambda_H}\right]$$

TCAS Horizontal Indicators

Tau

$$\tau = -\frac{r}{\dot{r}}$$

r: Slant Range

Estimation of time remaining to the Closest Point of Approach

Tau Modified

$$\tau_{mod} = -\frac{r^2 - DMOD^2}{r\dot{r}}$$

$DMOD$: Distance Modification

DMOD introduced to modulate a desired warning time for a sustained relative acceleration when the acceleration is normal the initial relative velocity vector.

“A New Threat Detection Criterion for Airborne Collision Avoidance Systems” RL Ford, DL Powell - Journal of Navigation, 1990

Encounter Model

Characteristics

- Origin: CPA point
- Initial positions of Ownship and Intruder depend on their speeds, pointing to the CPA.

$$\vec{r_1}(t_0) = -v_1 \cdot t_{CPA} \cdot \vec{i}$$

$$\vec{r_2}(t_0) = v_2 \cdot t_{CPA} \cdot (\cos(\theta) \cdot \vec{i} + \sin(\theta) \cdot \vec{j})$$

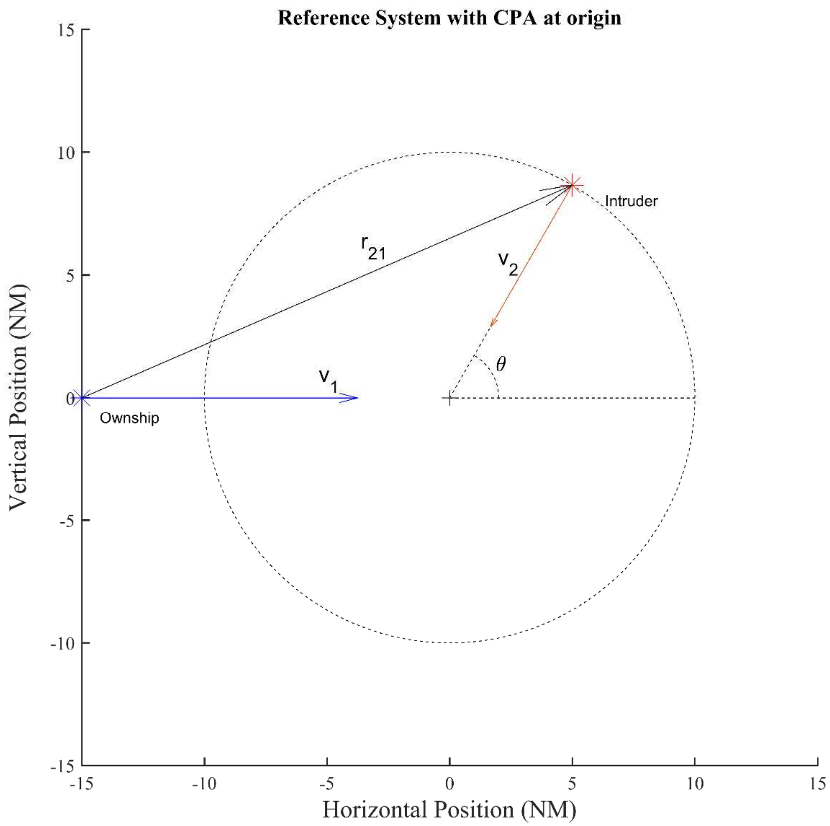
- Constant speed for both aircraft.

$$\overrightarrow{r_{21}}(t) = \overrightarrow{r_{21}}(t_0) + \overrightarrow{v_{21}}(t_0) \cdot t$$

$$\overrightarrow{r_{21}}(t_0) = t_{CPA} \cdot (v_2 \cdot \cos(\theta) + v_1) \cdot \vec{i} + t_{CPA} \cdot v_2 \cdot \sin(\theta) \cdot \vec{j}$$

$$\overrightarrow{v_{21}}(t_0) = -(v_2 \cdot \cos(\theta) + v_1) \cdot \vec{i} - v_2 \cdot \sin(\theta) \cdot \vec{j}$$

$$\|\overrightarrow{r_{21}}(t)\| = \sqrt{(v_1{}^2 + v_2{}^2 - 2 \cdot v_1 \cdot v_2 \cdot \cos(\theta)) \cdot (t - t_{CPA})^2}$$



TCAS Indicators Expressions

Tau

$$\tau = t_{CPA} - t$$

Tau Modified

$$\tau_{mod} = \frac{V(v_1, v_2, \theta) \cdot (t - t_{CPA})^2 - DMOD^2}{V(v_1, v_2, \theta) \cdot (t_{CPA} - t)}$$

$$V(v_1, v_2, \theta) = (v_1{}^2 + v_2{}^2 - 2 \cdot v_1 \cdot v_2 \cdot \cos(\theta))$$

$$(t_{CPA} - t_{tau\_mod}) = \frac{\tau_{mod}}{2} \cdot (1 + \sqrt{1 + \frac{4 \cdot DMOD^2}{V(v_1, v_2, \theta) \cdot (\tau_{mod})^2}})$$

Complexity Indicator in the Encounter Model

Complexity

$$CM_H(t) = \left(1 + \frac{\left(\frac{R}{2}\right)^2}{\left(\max\left(\frac{R}{2}, \sqrt{V(v_1, v_2, \theta)} \cdot (t_{CPA} - t)\right)^2\right)}\right) \cdot e^{\frac{T_{SM}}{(t_{CPA} - t)}}$$

$\frac{R}{2} > \|\overrightarrow{r_{21}}\|$

$$t_{IC} = \left(t_{CPA} - \frac{T_{SM}}{\ln\left(\frac{C_{mH}}{2}\right)}\right)$$

$\frac{R}{2} \leq \|\overrightarrow{r_{21}}\|$

$$\frac{CM_H \cdot V(v_1, v_2, \theta) \cdot (t_{CPA} - t)^2}{\left(\frac{R}{2}\right)^2 + CM_H \cdot V(v_1, v_2, \theta) \cdot (t_{CPA} - t)^2} - e^{\frac{T_{SM}}{(t_{CPA} - t)}} = 0$$

Solving Numerically → t_{IC}

Parameters

Encounter Model

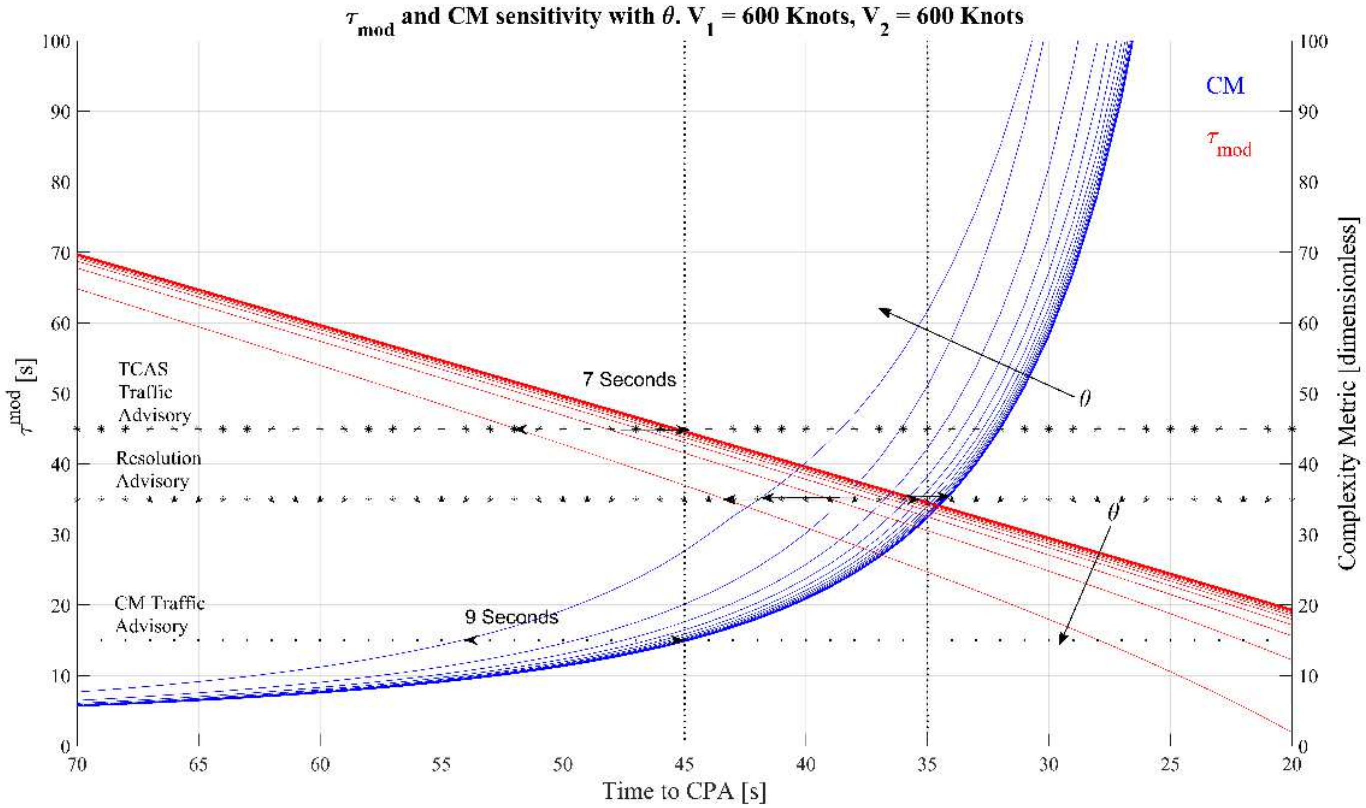
v_1, v_2 (0,600] Knots
θ [0, pi]
t_{CPA}=120 s

TCAS II

SL = 7
DMOD = 1.1 NM
RA = 35 s

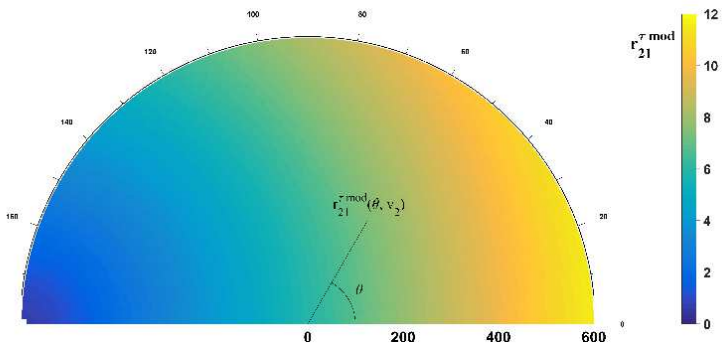
Complexity

R = 5 NM
T_{SM} = 120 s
CM_H= 35

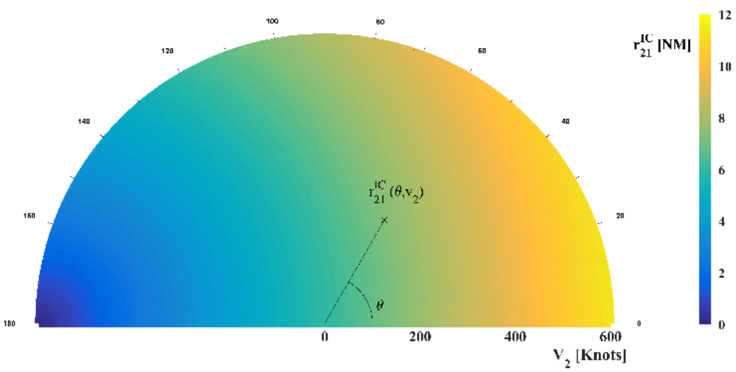


Range for triggering RA

$r_{21}^{\tau\ mod}(\theta, v_2)$ for v_1 = 600 Knots, SL = 7, τ_{mod} = 35



$r_{21}^{IC}(\theta, V_2)$ for V_1 = 600 Knots, T_{sm} = 120, R = 5NM

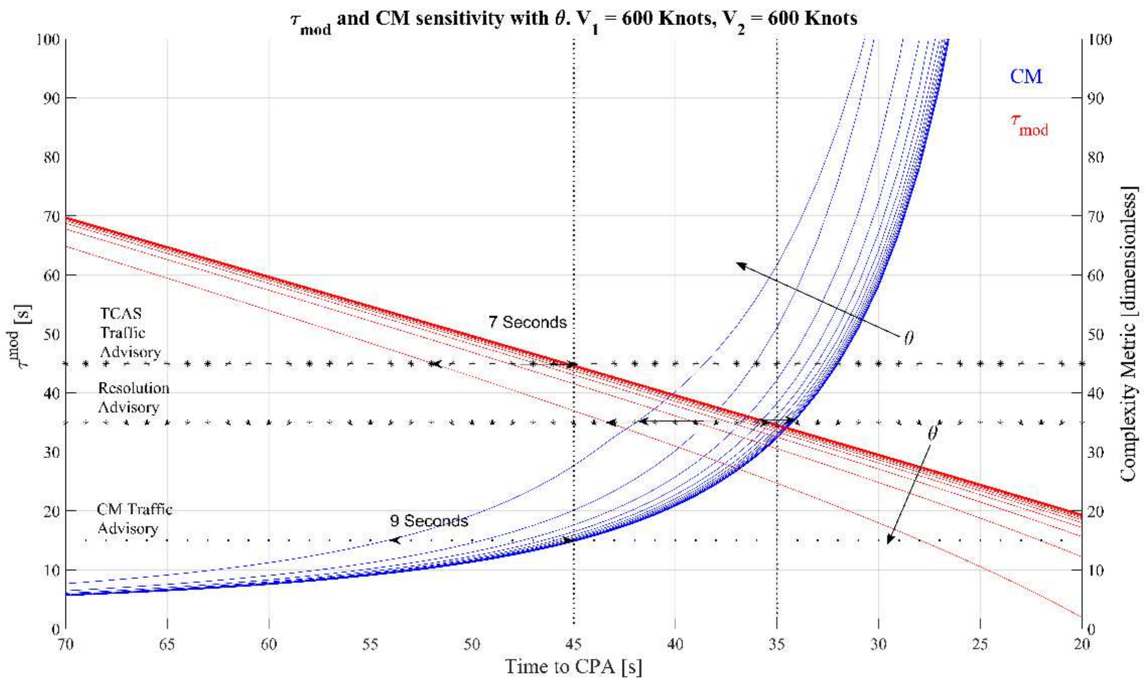


V2 [Knots]	Range [NM] for Tau, Tau_Mod, CM = 35. Theta = 0		
	tau	tau_mod	CM
300	8.75	8.78	8.63
400	9.72	9.75	9.55
500	10.69	10.72	10.48
600	11.67	11.69	11.4

V2 [Knots]	Range [NM] for Tau, Tau_Mod, CM = 35. Theta = 90		
	tau	tau_mod	CM
300	6.52	6.57	6.54
400	7.01	7.05	7.00
500	7.59	7.63	7.54
600	8.25	8.29	8.16

V2 [Knots]	Range [NM] for Tau, Tau_Mod, CM = 35. Theta = 180		
	tau	tau_mod	CM
300	2.92	3.02	3.24
400	1.94	2.09	2.33
500	0.972	1.22	1.17
600	0	~	0

Sensitivity to the encounter angle



- Warnings triggered earlier as the module of relative speed decreases because of the encounter angle.
- For both indicators, more sensitivity near alert thresholds.

Complexity

- Tsm is the Sensitivity Level parameter
- Upper and lower bounded depending on R.
- Less sensitive to relative speeds module than tau_mod.

Conclusions

- Complexity metric and TCAS τ_{mod} behave similarly in terms of:
  - range between aircraft when threshold for RA alerts are reached;
  - response trend to changes in the encounter angle.
- Complexity metric (exponential) is:
  - less sensitive to variations in the encounter angle and bounded by R;
  - difficult to establish a pre-alert (traffic advisory) due to its exponential behaviour, but TA would be triggered earlier for similar headings;
  - the CM could be adjusted with the characteristic time (Tsm).
- The CM could be adjusted to tend to TCAS horizontal indicators at the transition stage between SM and ACAS. Resolution actions based on the CM could remove non-coherent actions issued by different safety nets (if TCAS is considered as a fixed one)
- The CM is not intended to replace TCAS, but for managing multi-aircraft encounters based on a complexity criteria

Thank you! Any questions?