Probabilistic Aircraft Conflict Detection Considering Ensemble Weather Forecast

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1. Introduction
2. Problem Formulation
3. Results
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Motivation

A promising approach to improve current prediction and optimisation mechanisms towards meeting Single European Sky goals is the modelling, analysis, and management of the uncertainty present in ATM.

One of the main sources of uncertainty that affect the ATM system, as identified by the ComplexWorld Research Network, is weather uncertainty.

The objective of this work is to analyze the effects of wind uncertainty on conflict detection.
**Approach I**

- **Wind uncertainty**: provided by *Ensemble Weather Forecasts*; typically, a collection of 10 to 50 forecasts.

According to the IMET project, there are **two approaches** for trajectory prediction:

- Ensemble trajectory prediction
- Probabilistic trajectory prediction
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**Approach II**

- **Conflict detection:** between two aircraft, both flying at constant airspeed, course and altitude.

  The conflict is characterized by:
  - minimum distance \((d_{\text{min}})\),
  - time to minimum distance \((t_{d_{\text{min}}})\),
  - probability of conflict \((P_{\text{con}})\).

- **Analysis:** based on the **Transformation of Random Variables**, the wind probability density functions (PDFs) are evolved to obtain the PDFs and values of the conflict indicators.
Applicability

- Air-traffic **safety and efficiency** may benefit from the inclusion of weather uncertainty in automated conflict detection at all levels:
  
  - **Long range** (time horizon of several hours)
    Trajectories may be **strategically deconflicted** even prior to the take-off.
  
  - **Mid** (tens of minutes) and **short range** (seconds to minutes)
    Decision support tools and safety nets (e.g. MTCD and STCA) may notify the conflicts to the ATCOs according to their probability of occurrence, thus reducing the number of missed and false alerts.
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Assumptions

- A **North-East reference system** is used.
- Both aircraft, A and B, are affected by the **same wind** ($\vec{w}$).
- The wind components ($w_x$ and $w_y$) are **uncertain and statistically independent**.
- Both aircraft fly at **constant airspeed** ($V_A$ and $V_B$), **constant course** ($\psi_A$ and $\psi_B$), and the same **constant altitude**.
- The **initial positions** ($\vec{s}_0,A$ and $\vec{s}_0,B$), **airspeeds** and **courses** are **perfectly known**.
- The **initial separation** between aircraft is **greater than a separation requirement** $D$ (e.g., 5 NM) and **they are approaching**.
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Absolute motion of each aircraft

- **Position of each aircraft** at any time $t$:
  \[ \vec{s}_A(t) = \vec{s}_{0,A} + \vec{V}_{g,A} t, \]
  \[ \vec{s}_B(t) = \vec{s}_{0,B} + \vec{V}_{g,B} t. \]

- **Groundspeeds** $\vec{V}_{g,A}$ and $\vec{V}_{g,B}$ are obtained from the wind triangles:
  \[ \vec{V}_{g,A} = \vec{V}_A + \vec{w} = V_A \begin{bmatrix} \cos (\psi_A - \arcsin (w_{c,A}/V_A)) \\ \sin (\psi_A - \arcsin (w_{c,A}/V_A)) \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \end{bmatrix}, \]
  \[ \vec{V}_{g,B} = \vec{V}_B + \vec{w} = V_B \begin{bmatrix} \cos (\psi_B - \arcsin (w_{c,B}/V_B)) \\ \sin (\psi_B - \arcsin (w_{c,B}/V_B)) \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \end{bmatrix}. \]

**Crosswinds** $w_{c,A}$ and $w_{c,B}$ are positive if they are from the left wing:
  \[ w_{c,A} = w_y \cos \psi_A - w_x \sin \psi_A, \quad w_{c,B} = w_y \cos \psi_B - w_x \sin \psi_B. \]
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  $$\vec{V}_{g,B} = \vec{V}_B + \vec{w} = V_B \begin{bmatrix} \cos (\psi_B - \arcsin (w_{c,B}/V_B)) \\ \sin (\psi_B - \arcsin (w_{c,B}/V_B)) \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \end{bmatrix}.$$ 

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\[ w_{c,A} = w_y \cos(\psi_A) - w_x \sin(\psi_A), \quad w_{c,B} = w_y \cos(\psi_B) - w_x \sin(\psi_B). \]

The courses are known, but **the magnitudes of the groundspeeds are uncertain**.
Relative motion between the aircraft

- The conflict indicators of this work only depend on the relative motion between the two aircraft.

- Relative position between the two aircraft at any time $t$:

  \[
  \vec{s}(t) = \vec{s}_B(t) - \vec{s}_A(t) = \vec{s}_0 + \vec{V}_g t,
  \]

  where

  \[
  \vec{s}_0 = \vec{s}_{0,B} - \vec{s}_{0,A},
  \vec{V}_g = \vec{V}_{g,B} - \vec{V}_{g,A}.
  \]

  and the relative groundspeed $\vec{V}_g$ is given by:

  \[
  \vec{V}_g = V_B \begin{bmatrix} \cos(\psi_B - \arcsin(wc,B/V_B)) \\ \sin(\psi_B - \arcsin(wc,B/V_B)) \end{bmatrix} - V_A \begin{bmatrix} \cos(\psi_A - \arcsin(wc,A/V_A)) \\ \sin(\psi_A - \arcsin(wc,A/V_A)) \end{bmatrix}.
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  and the **relative groundspeed** $\vec{V}_g$ is given by:
  \[
  \vec{V}_g = V_B \left[ \cos \left( \psi_B - \arcsin \left( \frac{w_{c,B}}{V_B} \right) \right) \right] - V_A \left[ \cos \left( \psi_A - \arcsin \left( \frac{w_{c,A}}{V_A} \right) \right) \right] - V_A \left[ \sin \left( \psi_B - \arcsin \left( \frac{w_{c,B}}{V_B} \right) \right) \right].
  \]

The magnitude and direction of the relative groundspeed are uncertain and, under the hypotheses of this work, only affected by the crosswinds.
Conflict indicators

The distance between the two aircraft at any time, \( d(t) \), is the magnitude of the relative position:

\[
d(t) = \| \vec{s}(t) \| = \sqrt{s_0^2 + 2 \vec{s}_0 \vec{V}_g t + V_g^2 t^2}.
\]

- Minimum distance:
  \[
d_{\text{min}} = \sqrt{s_0^2 - (\vec{s}_0 \vec{V}_g)^2} / V_g^2.
\]

- Time to minimum distance:
  \[
t_{d_{\text{min}}} = -\left( \vec{s}_0 \vec{V}_g \right) / V_g^2.
\]

- Probability of conflict:
  \[
P_{\text{con}} = P[d_{\text{min}} \leq D].
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Conflict indicators

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These conflict indicators depend on the crosswinds through \( \vec{V}_g \) but not on the along track-winds.
Basis of the bivariate transformation:

- $w_x$ and $w_y$ are random variables with joint PDF $f_{w_x,w_y}(w_x, w_y)$, and $R$ is the set in the $w_x w_y$-plane where $f_{w_x,w_y}(w_x, w_y) > 0$.
- $v_1$ and $v_2$ are two random variables whose PDFs are to be found, and $v_1 = g_1(w_x, w_y)$ and $v_2 = g_2(w_x, w_y)$ define a one-to-one transformation of $R$ onto a set $S$ in the $v_1 v_2$-plane.
- If $w_x = h_1(v_1, v_2)$ and $w_y = h_2(v_1, v_2)$, then the PDF of $v_1$ and $v_2$ is given by

$$f_{v_1,v_2}(v_1, v_2) = \begin{cases} f_{w_x,w_y}(h_1(v_1, v_2), h_2(v_1, v_2)) |J| & \text{if } (v_1, v_2) \in S, \\ 0 & \text{otherwise,} \end{cases}$$

where $|J|$ is the absolute value of the Jacobian determinant

$$J = \begin{vmatrix} \frac{\partial h_1(v_1, v_2)}{\partial v_1} & \frac{\partial h_1(v_1, v_2)}{\partial v_2} \\ \frac{\partial h_2(v_1, v_2)}{\partial v_1} & \frac{\partial h_2(v_1, v_2)}{\partial v_2} \end{vmatrix}.$$
In this work, $v_1$ is any of the indicators $d_{min}$ or $t_{d_{min}}$, and $v_2$ is a dummy variable which has been chosen to be $w_y$.

The PDF of $v_1$ can be obtained by integrating $f_{v_1,v_2}$ in $v_2$

$$f_{v_1}(v_1) = \int_{-\infty}^{\infty} f_{v_1,v_2}(v_1,v_2) dv_2$$

The expected value, the typical deviation and the probability of $v_1$ being smaller than a given value are given by

$$E[v_1] = \int_{-\infty}^{\infty} v_1 f_{v_1}(v_1) dv_1,$$

$$\sigma[v_1] = \left[\int_{-\infty}^{\infty} v_1^2 f_{v_1}(v_1) dv_1 - (E[v_1])^2\right]^{1/2},$$

$$P[v_1 < a] = \int_{-\infty}^{a} f_{v_1}(v_1) dv_1.$$
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Scenario

- **Aircraft positions and speeds**
  \[ \vec{s}_{0,A} = [0, 0], \vec{s}_{0,B} = [18520, 18520] \text{ m}, \]
  \[ V_A = V_B = 240 \text{ m/s}, \]
  \[ \psi_A = 90 \text{ deg}, \psi_B = 135 \text{ deg}, \]
  \[ D = 9260 \text{ m (5 NM)}. \]

- **Wind**
  Same **uniform probability distribution** for \( w_x \) and \( w_y \)
  \[ f_{w_i}(w_i) = \begin{cases} 
  1/(2\delta_w) & w_i \in [\bar{w} - \delta_w, \bar{w} + \delta_w], \\
  0 & \text{otherwise}, 
\end{cases} \]
  where \( i \in \{x, y\}, \bar{w} \in [-20, 20] \text{ m/s}, \) and \( \delta_w \in [0, 25] \text{ m/s}. \)
  If \( \bar{w} > 0 \) the wind points Northeast and if \( \bar{w} < 0 \) it points Southwest (on average).
  Since \( w_x \) and \( w_y \) are statistically independent, the **joint PDF** is
  \[ f_{w_x,w_y}(w_x, w_y) = f_{w_x}(w_x)f_{w_y}(w_y) \]

- **Results** are **validated by the Monte Carlo method** (8.4 million samples).
Minimum distance I

- Results for $\bar{w} = 0$ and $\delta_{w} = 20 \text{ m/s}$:

$$E[d_{\text{min}}] = 10012 \text{ m}$$
$$\sigma[d_{\text{min}}] = 1076 \text{ m}$$
$$P_{\text{con}} = 28.0\%$$
Results for $\bar{w} \in [-20, 20]$ and $\delta_w = 5, 10, 15, 20, 25$ m/s:

$E[d_{\text{min}}]$ does not depend on $\delta_w$. In this scenario, it decreases as $\bar{w}$ increases because the wind changes from pointing Southwest to pointing Northwest.

$\sigma[d_{\text{min}}]$ increases as $\delta_w$ increases. Its dependence with $\bar{w}$ is very weak.
Time to minimum distance

- Results for $\bar{w} = 0$ and $\delta_w = 20$ m/s:

$$E[t_{d_{min}}] = 131.9 \text{ s}$$
$$\sigma[t_{d_{min}}] = 6.4 \text{ s}$$

- Results for $\bar{w} \in [-20, 20]$ and $\delta_w = 5, 10, 15, 20, 25$ m/s:

$E[t_{d_{min}}]$ is almost independent of $\bar{w}$ and $\delta_w$. $\sigma[t_{d_{min}}]$ increases as $\delta_w$ increases, its dependence with $\bar{w}$ is very weak.
Probability of conflict

- Results for \( \bar{w} \in [-20, 20] \) and \( \delta_w = 0, 5, 10, 15, 20, 25 \) m/s:

The certainty that a conflict does exist or does not exist decreases as the wind uncertainty increases.
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- The presented approach is capable of taking as input any type of wind distribution derived from Ensemble Weather Forecasts. The determination of the PDF from the ensemble forecast is an open challenge.

- Under the hypotheses that the aircraft follow a constant course and they are affected by the same wind, it has been found that the three considered indicators depend on the crosswinds seen by both aircraft, but not on the along-track winds.

- Numerical results have been presented for a particular scenario to show the potentiality of the proposed methodology.
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Future Work

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- Obtention of wind distributions from actual ensemble forecasts.
- Formulation of the problem for trajectories composed of segments with different courses.
- Aircraft affected by a different wind obtained from a statistically-correlated wind-field.
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Thanks! Questions?