Abstract—Recent works in the area of Complex Systems have addressed the robustness of networks such as power grids, social groups and the Internet. The robustness is evaluated against an external perturbation that can be different in nature depending on the particular network. For instance, the failure of power transmission lines that can trigger a nationwide blackout or a general shutdown of routers and the consequent connectivity loss. In this work, we introduce metrics inspired by Complexity Science to explore the robustness of the air transportation system in the US with respect to delay propagation. We use an agent-based model recently developed to simulate delay propagation and assess the effect of disruptions in the network. These disruptions are introduced as initial conditions and can affect single flights or full airports. The model is then run with and without disruptions and the outcome is compared to quantify the system robustness. Our results indicate that large hubs (in the sense of number of offered destinations) are more vulnerable to flight delays than small or medium sized airports. However, the impact in the whole network of delays initiated in an airport does not depend on whether it is a hub or not. We also detect a set of high impact flights and explore the drivers that generate these long tail extreme events.

I. INTRODUCTION

Among all the different means of transport, air transportation is the one that has experienced the fastest growth in the last century [1]. In 2013, the number of domestic and international air passengers summed up 3,023 millions worldwide [2] and it is expected to increase by 6% this year [3]. The rapid increase in demand comes at a high price, causing the transport network to become congested. It is therefore of great importance to understand the interplay between the various components of the system, in which demand and capacity are two sides of the same coin.

The intricacy and interaction between the elements that compose the air-traffic system clearly qualifies it as a Complex System. Complexity is not used just to refer to complicated phenomena within Science, it emphasizes the notion of emergent behavior at the system level that surges from the interaction between its components. During the last decade, the scientific community has extensively studied these systems under the light of Network Science. By these means the air-traffic system can be represented as a network whose vertices represent airports and its edges direct flights during a fixed time period [4], [5], [6], [7]. Several aspects of the air traffic network have been studied. The first works [8], [9] focused on a topological description of the network structure. The results showed a high heterogeneity in the number of connections that bear each node (the so-called degree of a node) and the traffic sustained by each connection, finding a non-linear relation between the node degree and the fluxes of passengers [8].

The Air Transportation Network can also be understood as the backbone where different dynamical processes take place. A story of notable success was the modeling and forecasting of disease spreading using air traffic data [10]. Furthermore, delay propagation dynamics has also been studied as a process that heavily relies in the interconnectivity pattern of the air transportation network [11], [12].

Robustness against external perturbations is an important feature of the networks, bringing together the system structure and dynamics. It could be defined as the system ability to continue primary functions after a perturbation occurs. Perturbations can be modeled in different ways. Reference [13] studied how the network is affected to the removal of a fraction of nodes. Under certain conditions this may produce the network fragmentation, therefore severely damaging the communications between its components. In this work, the authors distinguish between random removals (errors) and targeted attacks to its most connected nodes (hubs). While networks with heterogeneous degree distributions (scale-free networks) are robust against random errors they are likely to fragment into smaller clusters if a critical fraction of its hubs
is removed. As noted in Ref. [14] this can be understood as a percolation process, thus as shown in Ref. [15] it is possible to derive exact analytical solutions for node and edge percolation (removal of a fraction of edges). In networks where the dynamics play a crucial role an initial disturbance can trigger a cascade of subsequent failures [16]. Such is the case in power grids [17], [18] or air transportation networks [19], [20]. This dynamic effect is enhanced by networks coupled together, the so called multiplex networks [21], [19]; where the failure of elements in one network can lead to a branching process affecting elements of other networks in a recursive way.

In this paper, we tackle the problem of the air transportation network robustness using US performance data. Instead of a structural view, we focus here on the robustness of the system dynamics. In particular, we consider our model of delay propagation described in Ref. [11]. In this case, the initial disruption is given by one or several delayed flights (when considered the airport disruption) that later, as the flight operations continue, can spread and multiply producing a cascade of reactionary delays. We therefore consider the initial disruption as a primary delay [22], [23] and the subsequent cascade as reactionary delays [24], [25]. As shown in Ref. [11] this ripple effect is boosted by the network connectivity through the aircraft rotation and crew and passenger connections between flights. Based on these findings, we define metrics able to assess the robustness of the network when a delay impact is produced by an individual delayed flight or a congested airport. Given that the events in the model are fully traceable, we develop a cause-effect analysis allowing us to reconstruct the trees of reactionary delay [24].

II. DATA SOURCES & MODELING FRAMEWORK

A detailed description of the modeling framework is provided at [11]. The delay propagation model uses delay data obtained from the Bureau of Transport Statistics (BTS) [26]. In particular, the information was obtained from the Airline On-time Performance Data. The database is constructed with the information provided by the 18th largest US certified air carriers with respect to the domestic scheduled passenger revenues. The database includes flight information for roughly the 76% of the total number of scheduled flights in the US including for each of them several performance measures. However, for modeling purposes we only use the date of flight, scheduled and arrival times, aircraft and airline identification codes, flight origin and destination. Cancellations and diverted flights are not used in the model. In this sense, we do not take these flights into account and the days selected for modeling are those with a relatively low fraction of canceled and diverted flights. Our assumption is that the flight data resembles the a priori airlines’ schedules. Therefore, with the aircraft code and the spatio-temporal localization of the flights obtained from the data, we can reconstruct the aircraft itineraries and consequently the airline schedules throughout the day. Under these assumptions, we use data from the 13th of July 2012. This day showed a high level of congestion, which according to the news was not imputable to meteorological [27], technical or labour causes. In Figure 1 we reconstructed the US airport network (USAN) for the selected day. As mentioned in the introduction, network nodes represent airports while the edges direct daily flights between them.

The schedule for the day is used as the model input. Therefore the model reproduces the flight dynamics given by the real planning of the day. Hence, each agent (aircraft) is tracked using its identifier code with a temporal resolution of one minute until the schedule of the selected day is fulfilled. Obviously if there is no disruption (primary delay) regarding the planning, the day would be completed without any inconvenience. The flight fluxes are generated following three microscopic sub-processes that rule the agents’ reaction to each other and the system: aircraft rotation, flight connectivity and airport congestion. The rotation is the itinerary of each aircraft throughout the day, i.e. it goes from airport A to B and then to C following the schedule arrival and departure times. An aircraft rotation is completed when all the previous flight legs have been fulfilled sequentially. A flight is not considered finished while the aircraft is in the gate-to-gate phase, which comprehends the taxi-in, taxi-out and airborne time. During this phase, it is not possible to absorb any delay. Consequently, in the model, arrival and departure delay are the same. Whenever the aircraft is attached to the gate (turn-around phase) it is possible to reduce inbound delay provided that there is sufficient slack time to absorb it. In addition, in the turn-around phase and after the flight has arrived, the aircraft has to comply with a minimum service time for ground operations (in the model is set at 20 min).

With flight connectivity, we account for crew and passenger connections between flights of the same airline. We do not have information of passenger connections and, therefore, a stochastic mechanism to connect flights is implemented taking into account airport connectivity levels obtained from another BTS dataset. Namely, the annual fraction of connecting passengers for each airport were collected from the DB1B Ticket and T100 Domestic Market repositories. Importantly, we make the assumption that crew connectivity is closely related with

![Fig. 2. Example of a tree of reactionary delay with 4 levels. In this case the delay per flight diminish downstream. ρ is the reproductive number of each flight.](image-url)
passenger connections given the lack of information on this issue. Therefore, for modeling purposes, crew and passenger connectivity are integrated together under flight connectivity. Consequently, each flight (of the same airline) has a probability of connection proportional, with a factor $\alpha$, to the connectivity levels of each airport. $\alpha$ makes possible to modulate the effects of flight connectivity in the model. With this in mind, a connection is randomly chosen by considering flights of the same airline with a schedule arrival time within a time window of three hours prior to the scheduled departure time of the flight under consideration. A flight is able to depart if and if only its connections have already arrived. In the simulations for July 13 we use an $\alpha$ value of 0.29, obtained by fitting the model simulation results to the congestion levels observed for that day.

Finally, we assume airports to have a finite capacity. Airport capacity is measured as the scheduled airport arrival rate for each hour (SAAR) of the day. When a perturbation occurs, the demand at the airport may vary and the actual arrival rate can exceed the schedule rate. Whenever this happen the next incoming aircraft will have to wait in order to be served. A queuing protocol based on First-in First-served (common operating procedure in the US) is implemented in each airport. This process may produce congestion at the airport and propagate delays to flights of different airlines.

III. PROPOSED MEASURES FOR ASSESSING NETWORK ROBUSTNESS AND IMPACT OF DELAYS

One way of evaluating the response of the system to a perturbation, is by exploring the delay $D_i(t)$ induced in flight or airport $i$ in response to a primary delay in a flight/airport $j$, $D_j^0(t_0)$, at time $t_0$. Therefore, we can measure the response of an element of the system to an induced perturbation as:

$$S_{ij}(t|t_0) = \frac{dD_i(t)}{dD_j^0(t_0)}.$$

(1)

Following the previous definition, we can construct the response matrix $S_{ij}$ with each entry measuring how susceptible an element $i$ is to a perturbation in $j$ ($S_{ij}(t|t_0)$). Regarding delay propagation dynamics, a perturbation could be to delay all departing and arriving flights at an airport within a certain time period and, then measuring the delay generated at an airport throughout the day. To better understand the system’s response to perturbation, we begin by defining the delay impact of an airport $i$ in the system as:

$$I_i = \sum_{j=1}^{N} S_{ij}^T.$$

(2)

Due to the heterogeneities present in the system, the perturbation outcome strongly depends on the time of the day that it is generated. Therefore, an airport perturbed at a certain hour $t_0$ might not have the same consequences in another hour.

In addition, it is possible to revert the argument and measure how robust an airport $i$ is to a perturbation generated in airport $j$. A way of evaluating the robustness of $i$ -or lack of it, i.e. vulnerability- is by means of $S_{ij}$ as:

$$R_i = \frac{1}{\sum_{j=1}^{N} S_{ij}}.$$

(3)

Hence, the robustness of an airport captures its response to perturbations in other airports. A large value of $R_i$ indicates that the airport is very robust.

As a way of measuring the delay impact of a single flight, we make use of the concept of trees of reactionary delay as described in Ref. [24]. The perturbation starts by setting a primary delay of 1 hour to an initial flight. The tree can contain one flight if there is enough slack time in the subsequent flight legs or connections given by the schedule. If this is not the case the delay propagates following a cascade-like effect as it is shown in Figure 2. In this example the perturbation branches by delaying the departures of the connecting flights and legs. Because of the complex pattern of connectivity in the network each reactionary tree will have different characteristics. We are interested, in measuring the total impact ($I_i$) and the average reproductive number ($\rho_i$) or branching rate of the tree generated by flight $i$. These two measures give an idea of
the extent of the perturbation in the system. The reproductive number is defined as the number of flights delayed by a delayed flight. For instance, the initial flight with 60 min has $\rho = 3$ because it affects three more flights in the level immediately downstream. The average reproductive number encapsulates the effect of the branching process in a tree. The total impact measures the fraction of minutes generated by the branching process over the initial delay. It could happen that a tree might be large enough in number of levels but with relatively low impact because the were enough slack time in the schedule to absorb the delays. Because of the inherent causality of the tree, we can also capture the influence of 3 important system components: the destination airport, the arrival time and the airline of the perturbed flight. Worth noting is that both impact variants (airport and tree impact) are conceptually equal, one at the level of nodes and the other at the level of edges.

IV. RESULTS

A. System response to airport perturbations

We begin by evaluating the system response to a perturbation of one hour at each airport of the network. To do so, the model is run delaying all incoming and departing flights of each airport, one airport at a time, for each hour of the day. Figure 3 shows the results concerning airport impact and robustness as a function of the number of connection of each airport ($k$). Surprisingly, the robustness has a steep decline for the airports with largest degree (Fig 3A). This unexpected result evinces the vulnerability of network hubs. In other words, large airports (in number of connections) are strongly affected by perturbations originated throughout the system. This is not the case for the rest, as it is clear that the relation is almost flat. One might expect a priori that hubs are robust enough to absorb delays due to its excess of capacity, but this result clearly contradicts this vision. A plausible explanation could be because of the reinforcement caused by flights that repeatedly go to a spoke and then return to the hub, magnifying the delay on the hubs. Figure 3B depicts the impact that each airport has on the network, evincing that it does not depend on the node degree. It is important to say though, that while hubs strongly perturb the system generating a large amount of delayed flights, the induced total initial delay at the airport is also high. Therefore, the $I_t^i$ for network hubs is not as high as one might expect. In any case, what this result reflects is that there is no relation whatsoever with respect to the airport size (in number of connections). In principle, this suggest that there are more subtle effects in the dynamics that might be related with other system features. In the next section we explore other possibilities that may affect the system response, accounting for individual flight impact.

B. System response to flight perturbations

Following a top-down analysis we keep going further down into the microlevel features of the airport network. If impact is indistinguishable at the airport level may be individual flights produce very different responses. We explore the system response by randomly selecting a flight for each airline, airport and schedule arrival time combinations, thus we generate simulations for 7658 different initial perturbations. In Figure 4A we evaluate this possibility by plotting the cumulative impact distribution of the trees generated by 60 minutes delayed flights. As previously explained each flight might produce a cascading effect developing trees of reactionary delays. The broad distribution of tree impacts signals the heterogeneities present in the system with respect to flights. In this sense, there are a few highly impact flights among many low impact ones. But the distribution shows that highly impact flights are likely to occur. Figure 4B shows the probability distribution of the average reproductive number $\bar{\rho}$. Similarly, there are quite a few trees with $\bar{\rho} < 1$, the most probable are flights with no impact at all $\rho = 0$. Needless to say that the delay propagation is boosted by trees with average reproductive number larger than one. In addition there exist flights that produce a large cascade with $\rho > 2$.

Spotted how susceptible the system is to individual flights, we remain to understand what are the main characteristics of flight impacts. To do so, we analyze the tree impact with respect to the time of the day the perturbation starts and according to the airline the flight belongs to. Figure 5A depicts the tree impact that generates each perturbation in
relation to the airline of the initially delayed flight. The first thing to notice is that some airlines (code: 20366, 19790, 20398, 20304, 19393) are likely to produce high impact flights evincing a long tail of extreme events. In addition the average airline impact is slightly different from one another. The main reason for this result should be a combination regarding the airline’s pattern of connectivity and the planned slack time within flights. Figure 5B depicts the flight impact regarding the schedule arrival time of the first flight of the tree. Not surprisingly the impact decreases along the day because the temporal cone of events (downstream flights that could be affected by the perturbation) is smaller as the time passes. Nevertheless, hours with highest impact time happens in the morning, not in early hours because, although with larger temporal cone, they also have the lowest average reproductive number (Figure 5D). According to the results shown in Figures 5B and Figure 5D there is a trade-off between the temporal horizon and the reproductive number. While the cone of events decreases through time the connectivity increases, therefore impact in between is the highest. Finally, Figure 5C confirms that \( \bar{\rho} \) is closely related to the extent of the impact on the system of each airline with high dense schedules combined with high impact trees. However there some exceptions (code: 20437, 19790) where the reproductive number is relatively high compared to the impact measured. As mentioned before these are large trees but with low impact because there was enough slack time in the schedule to absorb the delays produced.

To further understand the drivers behind the long tail of extreme events, we compare the data for the top 100 highest impact flights, those generating the largest impact trees, with other 100 flights selected at random. Regarding the destination airport of the initially perturbed flights, 67 airports are the initial target for the highest impact trees while also roughly 68 airports are the destination of the randomly chosen 100 flights. This represents roughly the 23% of all possible airports for July 13 for the two sets and there is no statistical difference between both. On the other hand, when the airline and scheduled arrival hour of the initial flights are taken into account there is a noticeable difference when compared to the random case. With respect to the hour of the day when the perturbation starts, the top impact tree flights are concentrated in the early morning hours (only 34% of the all daily operational hours). On the contrary, when flights are randomly selected this percentage increases up to 67%. Most striking are the results regarding the airline that the initial flight belongs to. In this case the statistical difference almost triples, with 33% for the top impact trees (5 airlines) while
the randomly selected flights affect to 95% of the airlines. These results merely confirm what was observed in Figure 5, although we would have expected the destination airport to be another key aspect of high impact flights.

V. CONCLUSION

In summary, we have defined a set of measures to assess the susceptibility of the different elements that compose the air transportation system. We explore the response of the system by means of impact and robustness to perturbations at different system levels; namely, at the airport and at the individual flight level. We find that the airport impact on the system has no clear relation to airport size (number of different destinations of the flights leaving the airport). In addition, we show that hubs are more vulnerable to perturbations throughout the system than medium and small sized airports. Among other results, we explore the influence of the airline and time of the day of the initially perturbed flights. Results display a dependence on the airline the flight belongs to, specially with regards to high impact flights. Also perturbations that start in the morning have higher impact than those in the afternoon because of a larger temporal cone of events. However, perturbations that start in the early hours of the day are an exception because of a relatively lower average reproductive number. Thus, we can conclude that the interplay between the airline connectivity pattern and the time of the day are two major causes of the different behaviors encountered in dynamics of delay propagation at the individual flight level.

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