Optimal Delay Allocation under High Flexibility Conditions during Demand-Capacity Imbalance

A theoretical approach to show the potential of the User Driven Prioritisation Process

Sergio Ruiz, Laurent Guichard and Nadine Pilon
DATM/DSR/NET
EUROCONTROL Experimental Centre
Bretigny-sur-Orge, France
sergio.ruiz.ext@eurocontrol.int, laurent.guichard@eurocontrol.int, nadine.pilon@eurocontrol.int

Abstract—**The User Driven Prioritisation Process (UDPP) is a concept under development in SESAR with as objective achieving additional flexibility for Airspace Users (AUs), i.e., the ability of the ATM system to accommodate AUs' changing business priorities. More flexibility could result in better cost-efficient delay management during congested situations, with substantial reductions of operational cost impacts for AUs. Equity (in the sense that one AU's prioritisation does not negatively impact another AU) is the main constraint for UDPP. This paper contributes to explore the limits of flexibility beyond the current UDPP validated features. A User Delay Optimisation Model (UDOM) is presented to analyse the hypothetical case in which an AU has high flexibility to minimise its own global delay costs, having full freedom to transfer delay among its flights and to exchange flight sequence positions with other AUs. After imposing a constraint of equity (total AU's delay must remain the same), it is shown that: a) there is an optimal level of delay for each of the AU's flights; and b) such equity condition increases flexibility in the system.**

Keywords- air traffic flow management (ATFM), flexibility, hotspot, demand and capacity balancing (DCB), slot allocation, SESAR, user driven prioritisation process (UDPP).

I. Introduction

In order to maintain safety in the Air Traffic Management (ATM) system, the European Network Air Traffic Flow Management (ATFM) Function at Airports or En-Route imposes delays or other measures on certain flights before departure [1], [2]. It is well known that ATFM delay causes operational irregularities with important costs to the Airspace Users (AUs), airports and passengers [4][5], ATFM delays being one of many irregularities reducing the operational efficiency, but one over which airlines have little influence.

Profitability in air transport industry is very sensitive to cost variations (profit margins might be as low as 1-2%) [6], thus AUs would like further flexibility, i.e., the ability of the ATM system to accommodate AUs' changing business priorities, to reduce the *'impact of delay'* (cost of delay) during irregular operations.

Delay is used today as a key performance indicator (KPI) of ATM *capacity* (capacity to maintain safety in operations), and thus most of the KPIs steering the ATFM Demand and Capacity Balancing (DCB) Function are based on average delay per flight, while DCB targets are strongly oriented towards a 'No-Delay paradigm' [2], [3]. As a consequence, in the event of a demand-capacity imbalance (a.k.a., *hotspot*), the Flow Management Position (FMP) in charge will most likely find a solution that decreases the overall system delay first, and whenever possible also reduce the impact of delay on AUs.

However, the impact of delay on AUs' operations, which is highly important information only known by the AUs, cannot be fully taken into account by DCB. If AUs' priorities could be considered during the DCB decision-making processes, this will have a large positive impact on the efficiency and predictability of the ATM operations. Airspace Users' participation in ATM and airport collaborative processes is therefore essential to minimise the impacts of deteriorated operations on all such stakeholders, thus giving strong arguments for the application of de-centralised decision-making (i.e., user-driven approach) as potential solution to achieve efficiency in the ATFM slot/delay allocation [7], [8].

SESAR envisioned the development of the User Driven Prioritisation Process (UDPP) to achieve additional flexibility for AUs to adapt their operations in a more cost-efficient manner [9]. UDPP concept is today under development and new features are being progressively incorporated aiming to fulfil different operational requirements and implementation constraints. Some of these features have already been proposed and validated with different levels of maturity, such as Enhanced Slot Swapping validated in 2015 and deployed in May 2017. Other less mature features are explained in [9], [10].

The aim of this paper is to explore the limits of flexibility beyond the current UDPP validated features, in particular exploring the hypothetical case in which high flexibility is given to an AU to minimise its own global delay costs, i.e., the AU has full freedom to transfer its total *baseline delay* (i.e., initial ATFM delay) among its flights and to exchange freely flight sequence positions with other AUs only being subject to one particular equity constraint: AU's total baseline delay cannot be reduced.

The potential implications of introducing high flexibility subject to equity will be discussed via the theoretical analysis of the *User Delay Optimisation Model* (UDOM), a simplified mathematical framework developed in the context of this research to capture the complex relationships between time, cost of operations and the flexible and equitable allocation of slots and delay. Two different degrees of equitable flexibility will be explored: a) the equity condition must be fulfilled at each single hotspot; and, b) equity requirements can be fulfilled after many hotspots in a long-term period (e.g., one year).

The remainder of the paper has been structured as follows. Section II provides a background and state of the art of UDPP; Section III describes the UDOM framework and results; Section IV discusses the implications of such results for UDPP; Section V presents the conclusions and future works.

II. UDPP BACKGROUND AND DEFINTIONS

A. Current concept of operations and reason to change

In today's operations, a few hours before a potential demand-capacity imbalance is foreseen with a certain level of confidence, the Network Manager activates a *regulation* scenario and issues *'ATFM slots'*, which will apply a tactical time-based separation between flights to ease the safe and smooth management of air traffic flows and sector/airport capacities during tactical and flight execution operations [1], [2]. Those ATFM slots are then allocated to the flights involved in the regulation, thus changing their times of departure with respect to the original slots scheduled for those flights, and thus causing delay on flights as a consequence.

The ATFM slots are not allocated on an arbitrary basis. Instead the process typically follows a transparent set of rules and policies previously agreed and accepted by all the relevant ATM stakeholders, including the AUs. The most common policy used today to allocate delay –when no other more constraining rule or operational policy applies– is the First Planned First Served (FPFS), which sorts the flights by the estimated time of arrival at or over (ETA/ETO) the constrained airport or sector, according to the information present in the filed flight plans and assigns the slots in such order [2].

FPFS is widely accepted by AUs because it preserves the original sequence of flights (considered fair), and it is well accepted today in ATFM operations because it minimises the total delay in a regulation [11], [12]. FPFS policy does not take into account that delay is allocated differently to the flights and that each flight may have different impact of delay.

Figure 1 shows the cost model that is being developed in the context of UDPP together with the AUs participation. Each flight has its own particular complex cost structure only known to the AU. The cost structure of a flight is typically not linear, due to the presence of different milestones and time constraints for each flight, such as crew out-of-hours constraints, maintenance slot requirements (such as a ramp check), passenger missed-connection costs, high-yield passenger business-retention ('soft') costs, or a missed airport curfew, etc. (reference values for these and other variables affecting the cost of delay of flights and AUs can be found in [5]). If a flight is delayed so that these important milestones or constraints cannot be fulfilled, then large negative impacts on AUs operational costs are typically the consequence. To mitigate such impacts, the AUs would like whenever possible more flexibility to prioritise their flights to redistribute delay on the basis of the consequences on operations and costs.

AUs are very heterogeneous in their size, form and business strategies, and thus, they often have very different operational needs, in particular regarding the flights subject to ATFM regulations. But in general it has been recognised by AUs that flights often have some tolerance to delay (i.e., margins), because although a minute of delay always has a cost, this cost can often be considered marginal in practice if delay is not trespassing the more constraining operational margins.

Figure 2 shows three flights of the same AU that are impacted differently by delay, since each flight has a different position in a sequence as well as different cost structures. Note that each flight has very different cost structure shape, either in the size of their delay margins and/or in the magnitude of the impact of delay. In the example, flight FL001 has little delay and little impact of delay, FL002 has mid delay but relatively large impact, and FL003 has the largest delay but relatively small impact in comparison with FL002. Note that the impact of delay for a single flight (e.g., FL002) might also include the costs associated to the potential knock-on/cascade effect caused by a certain amount of delay allocated to that flight.

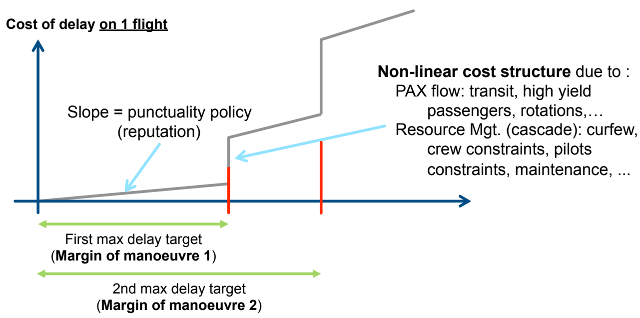

Figure 1. Typical cost structure model per flight

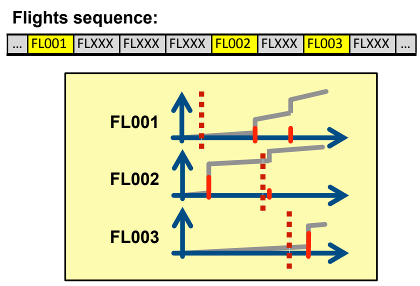

Figure 2. Different cost structures for each flight

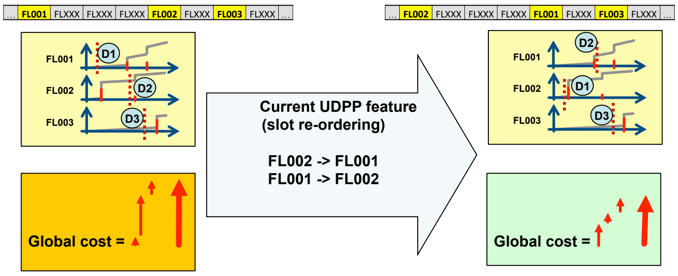

Figure 3. Contribution of flexibility to global cost optimisation

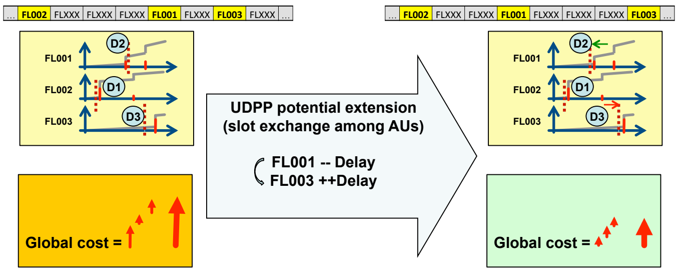

Figure 4. Advanced flexibility can bring even more benefits

Figure 3 shows the global cost of delay for the AU of the example taking into consideration his three flights. The initial situation in the baseline sequence (e.g., FPFS sequence) is shown in the left part of the figure. The right part of the figure shows the benefits of giving flexibility to AU to transfer delay between its flights. For instance, by exchanging the positions of FL001 and FL002 (UDPP Slot Swapping) the delay D1 initially allocated to FL001 is transferred to FL002, and delay D2 to FL001. A large cost reduction might be possible for the AU by just changing that position.

The UDPP mechanism of slot swapping has already been validated in terms of impact on equity and acceptability by the AUs and FMPs. Nevertheless, the AUs are not always in the ideal situation of having low priority flights (with enough margins and/or relatively low economic value) in positions nearby their most impacted (high priority) flights so they can exchange their positions between them. Indeed, in a recent study performed internally in EUROCONTROL (still not published), based on the analysis of all the airport regulations in 20 consecutive AIRAC cycles[1], it has been found that in 85% of the regulations in which AUs are involved, typically only a few flights (equal or less than 3) are affected, which strongly limits the flexibility provided by basic UDPP concepts such as slot swapping. In this situation in which the AUs have a small number of affected flights it is said that the AU is an LVUC (*Low Volume User in Constraint*) [10]. In addition, note that some AUs often operate just a few flights and thus they might have little flexibility or even never be able to prioritise their important flights, which might be inequitable from the *access* KPA point of view (e.g., business aviation is specially vulnerable to that problem). Therefore, there is a need to explore new features in UDPP to enable more flexibility for all the AUs, in particular LVUCs.

New UDPP features are being under research in the context of SESAR2020, including the possibility of exchanging slots and delay among different AUs, either in the same hotspot or in several hotspots over time. To shortly introduce the idea, in Figure 4 a new example is presented in which the output of the previous example (Figure 3) is the baseline situation. Note that the situation of the AU could notably improve if the flight FL001 could be advanced some positions in the sequence, thus reducing its delay. A new UDPP feature being investigated today is the possibility for flight FL001 to take a better position that was initially allocated to another AU (the position in front would be enough in this example). In exchange, the AU should compensate the delay reduction on that flight by accepting more delay in other flights (e.g., delay FL003 one or more positions), not necessarily in the same hotspot, which might be a good solution for LVUCs.

Such advanced concepts will increase a lot the flexibility (e.g., in Figure 4 the AU will benefit from a non-negligible cost reduction), but it becomes more complex to put in place a set of UDPP Rules for which equity can be demonstrated, i.e., ensure that the actions of one AU will not impact negatively the operational priorities of other AUs (equity is the main UDPP constraint).

Figure 5 shows a conceptual map of different degrees of flexibility being studied today in the UDPP context via the development of different models and features. The development of mechanisms that allow the AUs to exchange delay between themselves in a transparent and equitable manner is key to enhance flexibility and cost reduction opportunities. However, the introduction of highly flexible advanced mechanisms is constrained by the difficulties on designing, testing and validating, including the generation of proofs of equity. For instance, the extension of flexibility over time (exchange of delay between different hotspots) may require the introduction of a system based on 'credits' (a virtual currency with no monetary value) to account the amount of delay exchanged between flights and between AUs over time. This introduces additional challenges in terms of implementation, operational acceptability and access to include all AUs. Therefore, to make steps forward towards a more flexible ATM while minimising the associated risks, it is important that all relevant stakeholders (and specially AUs) understand the potential benefits of the different degrees of flexibility and equity potentially provided by each proposed UDPP mechanism/feature as well as its particular limitations.

Next sections will explore, through a simplified analytical approach, the effects that forcing equity in the UDPP mechanism may have on the AUs dominant strategies when they are able to transfer delay freely between their own flights, and in particular it will be shown how equity may contribute to increase, rather than to reduce, the levels of flexibility for AUs.

[1] Roughly, one year and a half from Jan 2016 to July 2017 (AIRACs from 1601 to 1707 taken from EUROCONTROL's DRR).

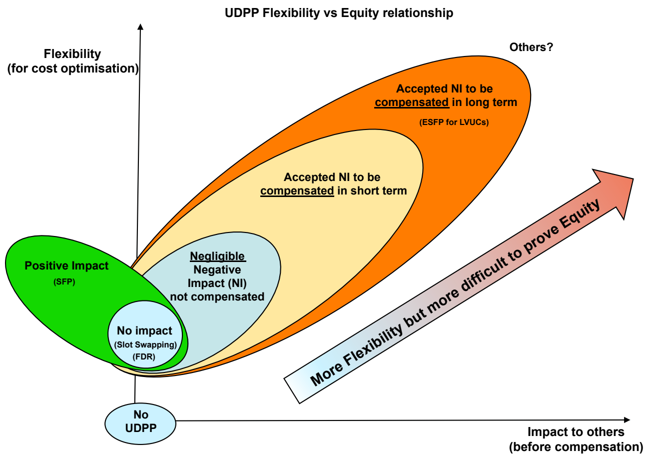

Figure 5. Conceptual map of Flexibility vs. Equity

III. *DESCRIPTION AND ANALYSIS OF THE USER DELAY OPTIMISATION MODEL (UDOM)*

TABLE I. DESCRIPTION OF MODEL PARAMETERS AND VARIABLES

Symbol	Range	Description
d	$[0,+\infty)$	*Delay* of a flight (d_0 means $d=0$)
U_0	$[0,+\infty)$	Max *utility* of a flight when its $d=0$
N	$[0,+\infty)$	Number of flights operated by an AU in the reference period
ε_i	$(-\infty,+\infty)$	*Elasticity* of the utility function of flight *i*. Used to model in a simplified way (continuous model) different operational flight margins
ρ_i	$[0,1]$	Probability of a flight *i* for being affected/delayed by a hotspot
δ_i	$[0,+\infty)$	*Average delay expected* for flight *i* in the route operated (i.e., typical delay from hotspots on that route)
$\tilde{d}_i$	$[0,+\infty)$	*Baseline (random) delay* for flight *i*
τ_i	$(-\infty,+\infty)$	*Delay shift,* to increase or reduce the delay of flight *i*
d_i^*	$[0,+\infty)$	*Optimal delay* for flight *i* in the actual hotspot
$d_i^{'}$	$(-\infty,+\infty)$	*Shifted delay*, i.e., delay difference between the optimal delay and the baseline delay, for flight *i*

Table 1 describes the model parameters and variables used in the UDOM model, which will be further discussed in the following sub-sections.

A. *Definitions and assumptions*

One of the main assumptions in UDOM is that the AUs taking part in the system can be modelled as utility maximising, i.e., the major objective of each AU is to maximise its utility function. Therefore, AUs are assumed to have a utility function, which is depending on several variables, such as the delay cost structure of each flight.

Utility is an important concept in economics and game theory, because it represents satisfaction experienced by the consumer of a good. In the context of this document, the concept of 'utility' will be understood as the value perceived by a particular AU if a given slot is allocated to a particular flight operated. Without loss of generality, in this document it is assumed that utility is directly related to economic profits obtained by the AUs for operating their flights, however the concept of utility may also include any type of operational constraints known by the flight dispatcher, and any indirect economic or non-economic type of benefits or costs.

Figure 6 shows a simplified representation of a utility function. In reality, utility functions are unknown and may be non-linear and non-convex (like for instance the cost functions). However, in this document it is assumed for the sake of simplicity that every single flight has a maximum utility when the delay, *d*, *is* zero, at a certain slot, and the utility is then progressively decreasing as far as the delay is increasing. Utility will be negative if the cost of delay has become greater than the maximum economic value expected for that flight if operated on time.

A utility function for a single flight, U(t), can be represented analytically as a quadratic function. If negative delay is not considered (simplification), the *utility* as a function of the delay, *d*, assigned to a flight can be expressed as:

$$
\begin{aligned}
&U(d) = \tfrac{\varepsilon}{2} d^2 + U_0 \qquad ,\forall d \geq 0 \\
&\frac{\partial' U}{\partial' d} = \mu_d = \varepsilon d \leq 0 \\
&\frac{\partial'' U}{\partial'' d} = \varepsilon < 0
\end{aligned}
\tag{1}
$$

, where ε is the elasticity of the utility function, μ_d the marginal utility and U_0 the maximum utility in case of no delay allocated to that flight. Note that since the elasticity is negative, any delay incurred by a flight will cause a reduction on the utility perceived by the AU for this flight. Different revenue and cost structures can be modelled by changing the parameters, thus they can be adapted to different types of AUs' type of activity (low cost, HUB, business aviation or others).

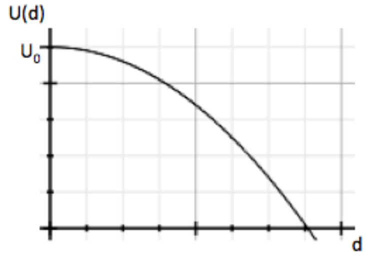

Figure 6. Utility function of a flight

Under the highly dynamic and uncertain ATM operational environment each flight operated is subject to a certain probability ρ_i of being involved in a hotspot and thus being delayed. In the long term (e.g., one year) the average delay expected for each flight can be quantified and expressed as δ_i. Therefore, the expected utility for an AU that operates N flights can be expressed as (d_0 means $d=0$, i.e., on-time):

$$\bar{U} = \sum_{i=1}^{N} U_i(d_0)(1-\rho_i) + \sum_{i=1}^{N} U_i(\delta_i)\rho_i \tag{2}$$

In the absence of uncertainty (i.e., in the hypothetical case in which all flights could be operated in their scheduled time), the maximum utility is determined only by the sum of utility functions of each individual flight operated by an AU (assumed constant daily utility). However, the actual utility is fluctuant and equal or lower to the maximum utility, thus the average long-term utility (i.e., expected utility) will be lower than the maximum in the absence of uncertainty. The higher the expected average delay for each operated flight and the higher the probability of being delayed, the lower the average expected utility will be. See Figure 7.

Participation in UDPP is voluntary. Thus, AUs will only participate if they do not receive a negative payoff, i.e., a lower expected utility than with no participation. In this section it is shown how AUs will be able to improve the expected return (increase the expected utility) of their operations in the presence of ATFM regulations that will affect their scheduled times, either if they decide to optimise in the short-term (i.e., only optimising the delay allocation between the flights involved in a particular hotspot) or if they decide to have a long-term strategy (i.e., optimising the average expected long-term utility by managing and allocating the delay between flights involved in hotspots occurring in different places and times). Such flexibility will create natural incentives to the AUs to participate, because they will typically be able to maximise their flight utility by managing and re-allocating the ATFM baseline delay of their own flights.

Since a short-term optimisation strategy can be considered as a particular case of the long-term optimisation strategy, the former will be explained after the second.

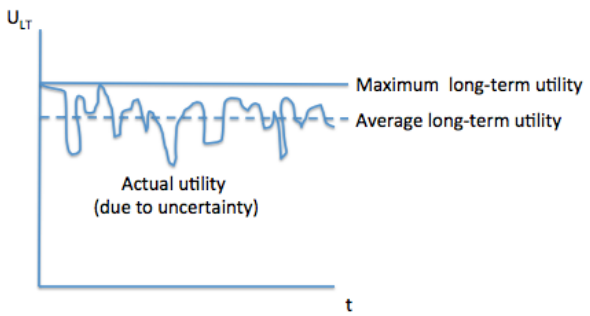

Figure 7. Expected utility with and without uncertainty

B. Definition and condition of the equity constraint

In the ideal high flexibility conditions under consideration, an AU would be able to exchange delay between his flights, even when the AU is a LVUC and has only one or a few flights in a given hotpot. Therefore, in case of a hotspot, the AU is willing to increase the delay with a *delay shift*, τ, for the flight that brings less utility in order to be able to reduce the delay in the same quantity for the flight that brings higher utility in the same or in future hotspots. To avoid potential system abuses, the model incorporates an equity constraint that forces the user to have no debts (nor surplus) at the end of the reference period (*AUs total baseline delay cannot be reduced*):

$$\sum_{i=0}^{N} \tau_i = 0 \tag{3}$$

C. UDOM with equity imposed at the end of a long-term reference period (multihotspot flexibilty)

To simplify the analysis of UDOM let us consider first a simple case in which an AU has two different flights, *f1* and *f2*, with equal probability ρ of being delayed and also equal expected delay δ (this could be the case, for instance, for two flights scheduled to the same destination airport during the same period). Then, the equation of expected utility according to (2) can be expressed as:

$$\bar{U}(d) = \left[U_{f1}(d_0) + U_{f2}(d_0)\right](1-\rho) + \left[U_{f1}(\delta) + U_{f2}(\delta)\right](\rho) \tag{4}$$

The optimisation problem that the AU faces, subject to the equity constraint, is:

$$\begin{aligned} &\max_{\tau_{f1},\tau_{f2}} \bar{U} = \left[U_{f1}(d_0) + U_{f2}(d_0)\right](1-\rho) + \left[U_{f1}(\delta+\tau_{f1}) + U_{f2}(\delta+\tau_{f2})\right](\rho) \\ &s.t. \left\{\tau_{f1} + \tau_{f2} = 0\right. \end{aligned} \tag{5}$$

or equivalently,

$$\max_{\tau_{f1}} \bar{U} = \left[U_{f1}(d_0) + U_{f2}(d_0)\right](1-\rho) + \left[U_{f1}(\delta+\tau_{f1}) + U_{f2}(\delta-\tau_{f1})\right](\rho) \tag{6}$$

Note that the value of $\delta+\tau$ for each flight will indicate to the AU which the optimal delay is for each of its flights subject to a hotspot with regard to the long-term average expected utility. To calculate the long-term optimum delay for each flight the AU must take into consideration all his flight operations expected for the reference period, together with the expected number of flights that might be regulated in the period as well as the average delay expected for each flight (statistical characterisation could be based on historical operational records).

The optimal delay shift, τ_i^*, can be found by equalling the first derivative of (6) to zero. For instance, for the flight *f1*:

$$\frac{\partial \bar{U}}{\partial \tau_{f1}} = \left[\varepsilon_{f1}(\delta+\tau_{f1}) - \varepsilon_{f2}(\delta-\tau_{f1})\right]\rho = 0 \tag{7}$$

$$\tau^*_{f1} = \delta \frac{\varepsilon_{f2} - \varepsilon_{f1}}{\varepsilon_{f2} + \varepsilon_{f1}} \tag{8}$$

The (long-term) *optimum delay* to apply to flight i in case of a hotspot is given by:

$$d^*_i = \delta + \tau^*_i \tag{9}$$

And the *shifted delay* d'_i to be applied when a flight i is affected by a random delay $\tilde{d}_i$ (that follows a distribution with mean δ) can be calculated with:

$$d'_i = \tilde{d}_i + \tau^*_i \tag{10}$$

1) Illustrative example 1: Multihotspot flexibility

Let us consider the following scenario for illustration: $\varepsilon_{f1} = -2; \varepsilon_{f2} - 10; \delta = 15\,\text{min}; \rho = 0.2$. Let us also consider a maximum utility per each of the flights equal to $U_0 = 500$. Therefore, the maximum total utility under zero uncertainty (i.e., $\rho = 0$) is given by (4), $U_{\max} = 500 + 500 = 1000$, while in the presence of uncertainty (i.e., $\rho = 0.2$) the expected utility is:

$$\begin{aligned} \bar{U} &= U_{\max}(0.8) + \left(U_{f1}(15) + U_{f2}(15)\right)(0.2) = \\ &= 1000(0.8) + \left(500 - \tfrac{2}{2}15^2 + 500 - \tfrac{10}{2}15^2\right) = 730 \end{aligned} \tag{11}$$

According to (8): $\tau^*_{f1} = 10; \tau^*_{f2} = -10$. This means that for a given random delay $\tilde{d}$ that affects flight *f1* or flight *f2* (as a consequence of an ATFM delay during a hotspot), the AU will try to apply an extra or reduced amount of delay to the flight until the (long-term) optimum delay given by (9) is reached. This has as effect that in the long-term the (optimised) expected utility will be, according to (6):

$$\begin{aligned} \bar{U} &= U_{\max}(0.8) + \left(U_{f1}(15 + \tau^*) + U_{f2}(15 - \tau^*)\right)(0.2) = \\ &= 1000(0.8) + \left(500 - \tfrac{2}{2}(15+10)^2 + 500 - \tfrac{10}{2}(15-10)^2\right) = 850 \end{aligned} \tag{12}$$

Figure 8 shows the comparison among the long-term utilities, i.e., without uncertainty (U=1000), with uncertainty and FPFS policy (U=730), and with uncertainty and the UDOM[2] sequence positions/delay allocation (U=850).

Analysing (8) it could be argued that any AU has economical interest to participate in this –ideal– UDPP mechanism if the difference between the elasticity of each flight utilities is different from zero ($\varepsilon_{f2} - \varepsilon_{f1} \neq 0$). Otherwise the AU will be indifferent (with UDOM the expected utility achieved would be the same as without UDOM).

[2] Under ideal flexibility conditions it can be assumed that all the exchange proposals are possible (i.e., no 'market incompleteness').

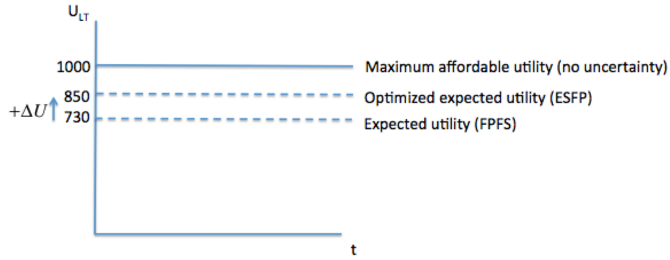

Figure 8. Long-term utilities for the scenario 1

D. Multiple hotspots with different probabilities to happen

The model depicted in (5) can be extended to consider the cases in which two flights have different probabilities to be involved in a hotspot, i.e., ρ_{f1} , ρ_{f2} , and when it happens they are delayed with different average delay, i.e., δ_{f1} , δ_{f2} (e.g., one flight has destination to Heathrow and the other to Madrid):

$$\begin{aligned} \max_{\tau_{f1},\tau_{f2}} \bar{U} &= \left[U_{f1}(d_0)(1-\rho_{f1}) + U_{f2}(d_0)(1-\rho_{f2})\right] \\ &+ \left[U_{f1}(\delta_{f1}+\tau_{f1})\rho_{f1} + U_{f2}(\delta_{f2}+\tau_{f2})\rho_{f2}\right] \\ s.t. &\left\{\tau_{f1} + \tau_{f2} = 0\right. \end{aligned} \tag{13}$$

For this model, the optimal delay shift for flight *f1*, τ^*_{f1} , can be found with:

$$\tau^*_{f1} = \frac{\delta_{f2}\rho_{f2}\varepsilon_{f2} - \delta_{f1}\rho_{f1}\varepsilon_{f1}}{\rho_{f2}\varepsilon_{f2} + \rho_{f1}\varepsilon_{f1}} \tag{14}$$

E. Generalisation of the UDOM to N flights and multiple hotspots

The optimisation model can also be generalised for N flights of a same AU, each of them characterised by a different elasticity ε_i and affected by delays with different probabilities ρ_i and with a particular and different expected delay δ_i :

$$\begin{aligned} \max_{\tau_1,\dots,\tau_N} \bar{U} &= \sum_{i=1}^{N} U_i(d_0)(1-\rho_i) + \sum_{i=1}^{N} U_i(\delta_i + \tau_i)(\rho_i) \\ s.t. &\left\{\sum_{i=1}^{N} \tau_i = 0\right. \end{aligned} \tag{15}$$

After some mathematical development (e.g., using multipliers of Lagrange), the optimal delay shift for each flight i can be expressed by:

$$\tau^*_i = \frac{\sum_{j=1}^{N} \delta_j}{\sum_{j=1}^{N} \frac{\varepsilon_i \rho_i}{\varepsilon_j \rho_j}} - \delta_i \tag{16}$$

Next example will be with three flights.

F. UDOM with equity imposed at the end of each hotspot (flexibility constrained by short-term reference periods)

In the particular case that high flexibility is only allowed to transfer delay among flights in the same hotspot (i.e., the AU must have the same total baseline delay at the end of the hotspot), the above formulae can be adapted to find the optimal solution for the AUs in this particular situation (note that optimising in the short-term may provide higher utility in the short-term but less in the long-term; however, long-term optimisation might be more heuristic, or perhaps impracticable, due to difficulties in assigning realistic costs to future flights). To optimise the delay allocation of the flights involved in the same hotspot, the AU just needs to substitute in equation (16) the value of the average delays, δ_i, by the actual (random) delay, $\tilde{d}_i$ while the probabilities for each flight to be involved in the hotspot should be parameterised to $\rho_i = 1$ (the flights are actually involved in a hotspot). The resulting optimal delay shift τ_i^* will determine the optimal delay for each flight in such a hotspot (see equation (10)) and therefore will indicate the demand of slots of such particular AU (note that the equity condition is fulfilled by the AU at the end of the short-term reference period, i.e., in a single hotspot, something that may not happen when the reference period is longer term, i.e., multiple hotpots).

1) Illustrative example 3: Single-hotspot flexibility

Consider an AU with three flights $f1, f2, f3$ involved in a hotspot. The actual delay allocated by FPFS is $\tilde{d}_{f1} = 5$, $\tilde{d}_{f2} = 12$ and $\tilde{d}_{f3} = 20$ respectively, and the elasticity of the utility functions is approximated by $\varepsilon_{f1} = -2$, $\varepsilon_{f2} = -10$ and $\varepsilon_{f3} = -9$. Consider the same maximum utility $U_0 = 500$ for all the flights (in case that they were not delayed). The total utility of the sequence corresponding with the Baseline Delay would be implemented (e.g., FPFS) can be calculated as:

$$U_{BD} = U_{f1}(5) + U_{f2}(12) + U_{f3}(20) = 475 - 220 - 1300 = -1045 \quad (17)$$

Using equation (16) it can be found that the optimal delay shifts are: $\tau_{f1}^* \approx 21$, $\tau_{f2}^* \approx -7$ and $\tau_{f3}^* \approx -14$. Applying these shifts, the new ESFP optimised utility for that AU is:

$$\begin{aligned} U_{UD}^* &= U_{f1}(5+21) + U_{f2}(12-7) + U_{f3}(20-14) = \\ &= -176 + 375 + 338 = +537 \end{aligned} \quad (18)$$

This illustrates that it is possible to move from large losses to significant profits (max utility without delay was 1500).

IV. Discussion

A. UDOM findings and contributions to UDPP

The UDOM analysis has shown that if high flexibility is given to an AU to re-allocate his flights' delay, with freedom to take and give delay from/to other AUs' flights, but constrained by equity forcing the AU to give back the delay that has been taken from the network (either in the short or the long term), then the AU shall be able to minimise his costs by transferring delay among his flights. This finding is very important, because it shows that, while equity can be enforced, all AUs shall have economic incentives to participate in UDPP.

The UDOM has also shown that, in the event of an ATFM hotspot in which a flight is delayed, the AU operating that flight may sometimes prefer to allocate more delay to that flight, even if this flight is of high relative importance[3]. This is something that may seem contradictory compared to the current 'no-delay' paradigm, but that is fully justified by the need of the AU to give back the delay taken from other AUs during the prioritisation of its flights (the AU must give positions in order to take positions). Such finding, i.e., the need of an AU to accept more delay in some flights to optimise his own utility, is also of high importance, since such self-interested participation of an AU, constrained by equity rules, is indeed what contributes to generate more flexibility for the others (beyond the flexibility already provided by the flexible utilisation of their own slots). When an AU voluntarily accepts more delay in a flight he is indeed offering positions in the sequence that can be taken by others to reduce their delay. Future UDPP mechanisms will therefore consist of a set of simple UDPP principles and rules for (almost) effortless cooperation between AUs driven by the enhancement of their own flexibility, while concentrating on the optimisation of their own operations.

B. UDOM limitations and UDPP Challenges

UDOM has been useful to facilitate the understanding of AUs' dominant strategies in a high flexible situation; however, the UDOM solutions are still far from being implementable in real operations. Hereafter some limitations are discussed (grouped in some well-known categories [13], [8]).

Externalities and Market Incompleteness: the decision-making processes of AUs are mutually dependent and subject to complex interactions. AUs trying to allocate their optimal delays may sometimes result in more than one flight pointing to the same position in the sequence, thus leading to the unavailability of the slot desired for some flights. Such type of negative externality (which could also be understood as a kind of *market incompleteness*) requires further research, e.g., to know *i)* how often AUs can actually reach their preferred solutions, *ii)* how often many UDOM de-centralised solutions could be merged into feasible and equitable sequence solutions, *iii)* how often other constraints, for instance availability of airport resources, is limiting flexibility of AUs, etc.

Bounded Rationality: in decision-making, individual rationality is limited by the available information, AUs' cognitive limitations and the finite time they have to make a decision. ATM environment is highly uncertain and dynamic, which poses a real challenge for the flight dispatchers that have

[3] E.g., an 'important' flight could receive 5 minutes of extra delay, if its baseline delay is 7 minutes and it has a delay tolerance of 15 minutes.

to make complex operational decisions with high impact on costs. In addition, the assignation of realistic costs to delays is often non trivial even for the AU himself. Ideal conditions of UDOM should be updated to shed light on such complexities of real operational environments.

Asymmetric Information and Incentive compatibility: the economic incentives of flexibility are a strong argument in favour of long-term cooperation (see [14], [15], and [16]). However, information is imperfect and it is asymmetrically available to AUs, which may lead to some AUs to be suspicious about the potential non-cooperative behaviour of others. Monitoring and protecting against potential abuses under high flexibility conditions, including data security and confidentiality (e.g., AUs learning other AU cost priorities from UDPP activity) require further research and validation.

V. CONCLUSIONS AND FUTURE WORK

A hypothetical situation has been studied in this paper with the help of the analytical model UDOM, in which an AU can optimise his operations under high flexibility conditions subject to one equity constraint: within a given reference period, the AU is allowed to re-allocate his ATFM baseline delay among his flights without reducing it. Such study contributes to illustrate the potential benefits and challenges of increasing flexibility with respect to current UDPP validated levels as well as to pave the way for future UDPP features.

Our analysis shows that, under UDOM ideal conditions, there is an optimal level of delay –typically greater than zero-for each flight affected by an ATFM slot, that minimises the total cost of flights operated by the AU within the period. Imposing equity constraints encourages AUs to request additional delay for some of their flights in order to reduce delay in other flights. This may be an important source of flexibility for other flights located later in the sequence (flights of the same AU or others'), because it enables a potential exchange of flight sequence positions (delay transfer) between flights with different delay needs.

This is an important finding that has two corollaries. Firstly, flexibility can be generated by and for UDPP participants by exploiting the available flight delay tolerances, either from own or from others' flights participating. Secondly, the larger the number of AUs participating in UDPP, the higher the levels of flexibility achievable for all participants (assuming that some flights tolerate extra delay at relatively low cost impact).

New UDPP features that allow the exchange of slots between AUs (to increase flexibility) will therefore consist of a set of simple UDPP principles and rules to smoothly coordinate between AUs (collaborative decision-making), to increase flexibility and equity on their own behalf (possibly benefiting also passengers and airports), while AUs can basically focus on optimising their own operations. It does not matter if AUs participate in UDPP only because of self-interest, because when their self-interested operational decisions are constrained with equity rules, their participation can nevertheless contribute to enhance the KPAs of flexibility, operational efficiency and cost-effectiveness in the overall ATM Network.

New UDPP features exploring the high degree of flexibility shown in this paper are difficult to develop and validate. Future research must firstly find a valid set of rules to solve those cases in which several AUs compete for the same slots; strong evidence will be required to prove that the new feature solutions always converge to feasible and equitable sequences; access and equity for all AUs must be guaranteed, including the incorporation of LVUCs needs; finally, realistic operational conditions, with highly uncertain and dynamic ATM constraints, must also be addressed in future UDPP research.

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