Probabilistic Traffic Models for Occupancy Counting

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***Abstract*—Air Traffic Management is subject to many uncertainties. These uncertainties drastically reduce overall predictability and force the introduction of margins having a negative impact on the capacity of the system.**

Moreover, as of today, uncertainties are not explicitly accounted for and human operator judgement and experience is relied upon to assess the "quality" of the estimations provided by the support tools.

One way to make it explicit is to convey uncertainty through probability distributions. Building on this approach, the paper describes how to derive probabilistic traffic models from historical data. These models are used as input to the algorithm developed by Gonze et al. [1] in order to compute occupancy count distributions. The application of the approach to one sector of EUROCONTROL's MUAC airspace is presented to show how uncertainty is captured by the proposed models.

I. Introduction

Air Traffic Management incurs complex operations involving numerous actors and processes carrying many unknowns and uncertainties. Today, these uncertainties (e.g. exact take-off time, route changes...) are only taken into account in the system in very limited ways and expert experience and judgement are relied upon to cater for them, often leading to bigger margins or conservative capacity estimates.

One way to make uncertainty explicit is to use probability distributions. Early 2016, the SESAR 2020 Exploratory Research project COPTRA has been started. COPTRA's main objective is to research ways to explicitly account for uncertainty in trajectory and traffic predictions using probabilistic trajectories and traffic situations.

Predicting occupancy counts is central to ATC planning and Demand-Capacity Balancing: the predicted values are used to chose the right airspace sectorisation or decide on necessary regulations. Today, however, the uncertainties on the inputs of counting process (like the take-off time) make the count predictions highly volatile. Following COPTRA's objective to make these uncertainties explicit and manageable, this paper describes an approach to attach uncertainty figures to sector entry and crossing times using historical data. Probabilistic sector sequences built from these figures are then used to compute occupancy count distributions using the algorithm detailed in [1].

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In Section II, traffic uncertainty is further discussed and detailed. Section III describes how COPTRA proposes to model probabilistic trajectories and to combine them to compute occupancy count distributions. The way this trajectory model is used to represent traffic demand uncertainty is presented in Section IV. In Section V, the approach is applied to MUAC's EDYYB5KL sector. Results and on-going work are presented in Section VI.

II. Traffic uncertainty

Many sources of uncertainty exist in ATM leading to non-optimal preventive actions (increased margins or buffers). In [2], Irvine details the *Capacity buffer theory* which states that sector capacity is set to control the probability of occupancy counts exceeding the peak acceptable level. The theory establishes a direct link between count uncertainty and sector capacity. This stresses for the need to better manage uncertainty and to find ways to make uncertainty explicit.

One major source of uncertainty affects the actual take-off (or more precisely, in the frame of this study, the off-block) time of the fight. Off-block and take-off can be delayed for numerous reasons. Delay data is extensively collected and documented (see e.g. EUROCONTROL's Central Office for Delay Analysis [3]).

When predicting sector occupancy counts, uncertainty also comes from the differences between the flight planning information (used to predict sector occupancy) and the way the flights are eventually flown. Figure 1 and 2 show, for a full day of the MUAC Brussels airspace, respectively the planned traffic and the actual traffic. Sources of these differences include the actions taken by ATC to deconflict traffic, weather or turbulence avoidance, emergency or flight diversions...

III. Combining Probable Trajectories

A. Probabilistic Trajectory Model

In order to cope with the uncertainties just outlined, COPTRA developed a probabilistic trajectory model able to take into account:

- Flight plan or route uncertainty: filed flight plans have to follow rules (like route network and route opening

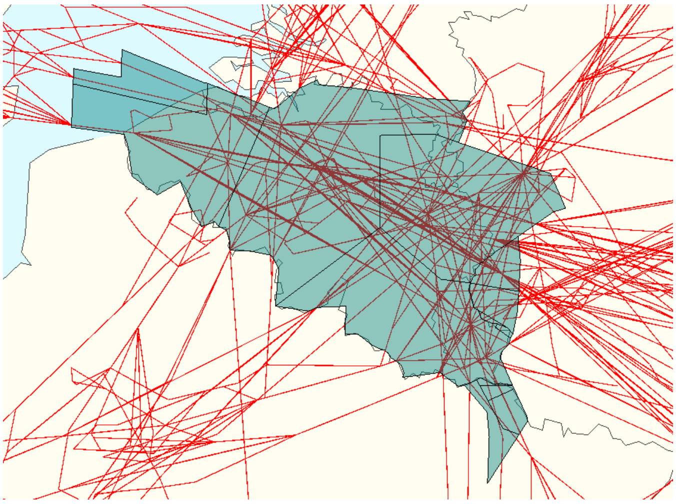

Fig. 1. Filed traffic (13/10/2016 — MUAC Brussels airspace)

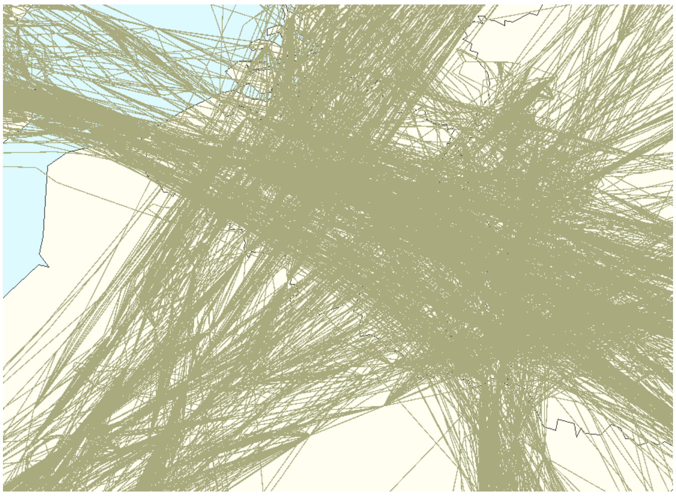

Fig. 2. Actual traffic (13/10/2016 — MUAC Brussels airspace)

schemes) which are potentially relaxed or changed during execution (e.g. "[Fly] direct" instructions given by controllers).
- Flight execution uncertainty: even with the flight route fixed, many inputs to the trajectory prediction process are not fully known or remain uncertain (weather, aircraft performance...) and lead to uncertainty on the flown trajectory.

A flight is then represented by a series of routes each having a given probability. A *probable route* is made of a series of points whose attribute values (longitude, latitude, altitude, time...) are expressed as probability distributions.

For the problem at hand, i.e. computing occupancy count distributions, the probable routes have been simplified to probable sector sequences: The probable route becomes a list of sectors crossed by the flights along with a probability distributions of the entry and exit times. In this model, the entry and exit time distributions are approximated by Gaussian distributions.

A probabilistic trajectory T_f for flight f can then be described formally as follows:

$$T_f = \left(p^*_{(f,i)}, \left(\text{es}_{(f,i,j)}, \mu^e_{(f,i,j)}, \sigma^e_{(f,i,j)}, \mu^l_{(f,i,j)}, \sigma^l_{(f,i,j)}\right)_j\right)_i$$

with i ranging from 1 to n_f for the different sector sequences and j ranging from 1 to $m_{(f,i)}$ on the sectors making the i^{th} sequence. f identifies the flight, $\text{es}_{(\ldots)}$ is the identifier of the crossed sector, $\mu^e_{(\ldots)}$ and $\sigma^e_{(\ldots)}$ are the parameters of the entry time distribution while $\mu^l_{(\ldots)}$ and $\sigma^l_{(\ldots)}$ parametrize the exit time distribution.

B. Computing occupancy count distributions

In [1], Gonze et al. describe a polynomial time algorithm to compute occupancy count distributions from probabilistic sector sequences following the model just described. An outline of the algorithm is given here. The reader is referred to [1] for details.

Given a set of probabilistic trajectories, the algorithm proceeds in two steps:

- For each flight trajectory, sampling time and sector, the probability $p_{(f,s,t)}$ of the flight being in the sector at the given time is computed.
- From these probabilities, the occupancy count distributions $\Theta_{(s,t)}$ are computed for each sector and sampling time.

These two steps are now detailed:

1) "In sector" probability: The probability that flight f is in sector s at time t, when following trajectory i is

$$p_{(f,s,i,t)} = P[t^e_{(f,s,i)} \leq t] - P[t^l_{(f,s,i)} \leq t]$$

where $t^e_{(f,s,i)}$ is the flight entry time in sector s when following the i^{th} trajectory. Similarly $t^l_{(f,s,i)}$ is the exit time. In other words $p_{(f,s,i,t)}$ is the probability that, at t, f has entered the sector but not left it yet.

As flight f may follow trajectory i with probability $p^*_{(f,i)}$, the probability having f in s at t, no matter which trajectory it follows is:

$$p_{(f,s,t)} = \sum_{i=1}^{n_f} p^*_{(f,i)}\; p_{(f,s,i,t)}.$$

2) Occupancy count distributions: The occupancy count distribution,

$$\Theta_{(s,t)} : \mathbf{N} \rightarrow [0,1] : k \rightarrow \Theta_{(s,t)}(k)$$

gives the probability of having $n \in \mathbf{N}$ flights in sector s at time t. It is the convolution of binomial distributions giving the probability of having flight f in sector s at time t with probability $p_{(f,s,t)}$. By standard methods computing $\Theta_{(s,t)}$ has an exponential computational cost.

However, by ordering the flights (f_j with $j \in 1...m$) and defining $q_{(i,j)}$ as the probability that, amongst the j first flights, there are i flights in sector s at time t, we have

$$\Theta_{(s,t)}(k) = q_{(k,m)}.$$

$q_{(i,j)}$ can then be defined using the following recurrence relation (for fixed s and t)

$$q_{(i,j)} = q_{(i,j-1)} \left(1 - p_{(f_j,s,t)}\right) + q_{(i-1,j-1)}\, p_{(f_j,s,t)}$$

which says that the probability of having i flights amongst the j first ones is the sum of

- The probability of having i flights in the sector amongst the $j-1$ first flights and not having flight j in it, and,
- The probability of having $i-1$ flights in the sector and having flight j in it.

Using this recurrence relation and applying dynamic programming techniques, the $\Theta_{(s,t)}$ distribution is computed in polynomial time.

C. Required inputs

To feed the algorithm computing the $\Theta_{(s,t)}$ occupancy count distributions, it is required to determine for each subject flight (f):

- The list of probable sector sequences and their respective probability: n_f and $p^*_{(f,i)}$
- for each sector sequence, the list of $m_{(f,i)}$ sectors crossed ($\text{es}_{(f,i,j)}$) and the entry and exit time distributions. As these distributions are assumed Gaussian, they are fully specified by their means and standard deviations: $\mu^e_{(f,i,j)}$, $\sigma^e_{(f,i,j)}$, $\mu^l_{(f,i,j)}$, $\sigma^l_{(f,i,j)}$.

Hereafter an approach is described that uses historical data to derive these distribution parameters.

Other ways to compute these inputs are possible and the COPTRA project also explores how to get them using probabilistic trajectory prediction.

IV. Probabilistic Traffic Model

The overall objective of the approach that will now be described is to compute the uncertainty attached to a given flight plan by deriving a series of probable sector sequences and the corresponding entry and exit time distributions. To that effect a *probabilistic traffic model* is first built from historical data. The model is then used to compute probabilistic sector sequences that are fed into the algorithm described in subsection III-B to get the occupancy count distributions $\Theta_{(s,t)}$.

The full traffic model is composed of sector sequence, entry time and crossing time distributions. In this paper, we present models for entry time and crossing time distributions. Sector sequence modelling is currently work in progress.

A. Historical data

EUROCONTROL maintains a repository, the Demand Data Repository or DDR2 [4], accessible to the ATM research community. Amongst others, the repository stores, in different formats, all the flights for the last five years.

The model described hereafter was built from the traffic crossing EUROCONTROL's MUAC Brussels airspace during three consecutive AIRAC[1] cycles (1607, 1608 and 1609) as stored in so-called AllFT+ formatted files. However, as the airspace of interest and the input data are parameters external to the model building process, models could be built from any historical dataset for any airspace of interest.

The input dataset contains 244108 unique flights crossing MUAC's Brussels airspace (EDYYB). For each flight in this dataset, the following features are extracted or computed:

Feature	Ref.
Departure airport	ADEP
Destination airport	ADES
Callsign	CS
IFPL Identifier	IFPLID
Week of the year	WOY
Day of the week	DOW
Hour of the day	HOD
Entering EDYYB from	FROM
Leaving EDYYB to	TO
Sector	ES
Delta off-block time	DOBT
Delta entry time	DETI
Crossing time	XGTI

This feature set is designed to capture most relevant traffic characteristics and to account for both temporal (yearly, weekly or daily) and spatial (routes or flows) trends. While the probabilistic trajectory model uses "absolute" entry and exit times, when building the traffic models, the sector entry time is computed as the delta from the actual off-block time. Similarly the time difference between the flight time of entry in the sector and the time of exit form the sector (sector crossing time) is used instead of the exit time. These two features allow to build models which are independent of the actual time at which the flight took place while permitting, for a subject flight, to compute entry and exit times distributions from the predicted or actual off-block time.

From the processed dataset, empirical entry time and crossing time conditional distributions can be derived for each sector. Figure 3 and Figure 4 show respectively typical entry time and crossing time distributions for a specific sector. The entry time distribution is conditioned on the departure airport. The crossing time distribution is unconditioned: it shows the crossing time frequencies for all the flights in the data sample.

The distributions illustrated are not Gaussian, as it is the case for to the vast majority. Modelling entry times or crossing times with Gaussian distributions would lead to an important loss of information: For instance, the multi-modality of the crossing time distribution probably accounts for the different routes crossing the elementary the elementary sector. Conditioning the distributions (e.g. on the route) might be a way to "separate" these different distributions. However, it might quickly lead to very specific models. This is the classical "bias-variance" trade-off (see e.g. [5]). Moreover, as it will be seen, adding conditions also increases the size of the model exponentially.

It was then decided to look for models that could represent any of these distributions (conditioned and unconditioned)

[1]Significant revisions in Aeronautical Information Publication are made every 28 days. This period is called AIRAC (Aeronautical Information Regulation and Control) cycle.

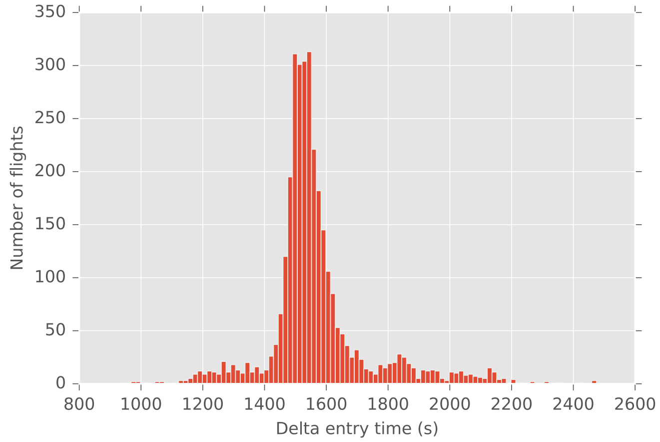

Fig. 3. Entry time distribution for traffic from EHAM (MUAC sector EDYYB37H)

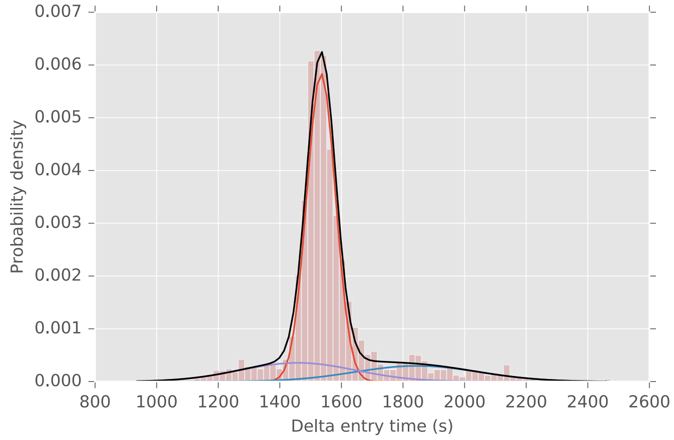

Fig. 5. EDYYB37EH entry times from EHAM fitted with a 3 components GMM

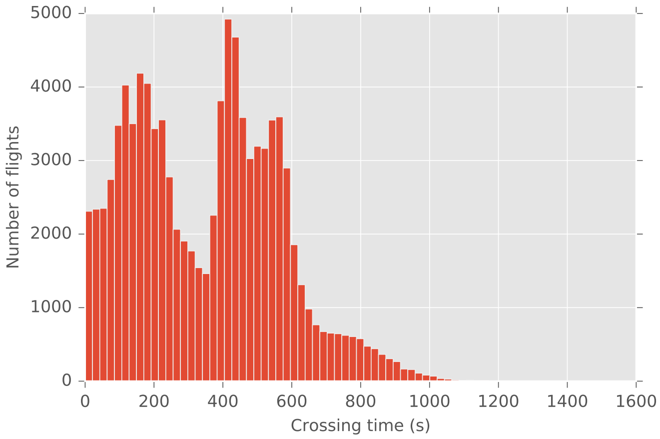

Fig. 4. Crossing time distribution for MUAC sector EDYYB37H

with sufficient accuracy. This would allow to explore models with a varying number of conditional parameters and start with simple models that could be enriched if necessary.

The use of Gaussian Mixture Models (GMM) is described hereafter. An additional and important advantage of GMM is that they can be used with the probabilistic trajectory model described in subsection III-A.

B. Gaussian Mixture Models

A Gaussian mixture model is a probabilistic model that assumes that all the data points are generated from a mixture of a finite number of Gaussian distributions with unknown parameters ([5], [6]).

$$\text{GMM} \sim \sum_{i=1}^{n} w_i N(\mu_i, \sigma_i)$$

with the corresponding probability density function:

$$p_{\text{GMM}}(x) = \sum_{i=1}^{n} w_i N(x|\mu_i, \sigma_i)$$

where w_i is the weight of the i^{th} Gaussian of the mixture with $\sum_{i=1}^{n} w_i = 1$. Similarly μ_i and σ_i are, respectively, the mean and the standard deviation of the i^{th} Gaussian. The resulting distribution would "generate" elements distributed along the Gaussian $N(\mu_i, \sigma_i)$ with a probability of w_i. In its general setting, GMM supports multivariate Gaussian distributions. Univariate distributions are sufficient here.

The process of fitting a GMM to given data is an unsupervised learning problem. To fit the GMM to the data, the unknown parameters n, w_i, μ_i and σ_i for i ranging from 1 to n will be determined from the historical dataset.

For a given n, these parameters can be determined using expectation-maximization or Bayesian techniques. Bayesian Information Criterion (BIC) can help to select the value of n [7].

In the frame of this work, expectation-maximization as implemented in the Python *Scikit-learn* toolbox [6] was used to fit the GMM. As BIC led to high values for n, visual inspection was used to determine the quality of the fit while favouring models having a small number (2 or 3) of components. Figures 5 and 6 show the GMM fitted to the empirical distribution of figures 3 and 4 respectively.

Both the entry times from Amsterdam and the crossing times were fitted by expectation-maximization with 3 components GMM. The parameters for the delta entry time GMM are:

i	w_i	μ_i (s)	σ_i (s)
1	.68	1534.08	46.64
2	.15	1844.98	202.37
3	.17	1458.48	187.52

while the parameters for the crossing time GMM are:

i	w_i	μ_i (s)	σ_i (s)
1	.38	138.54	73.22
2	.41	451.23	111.10
3	.21	568.11	198.81

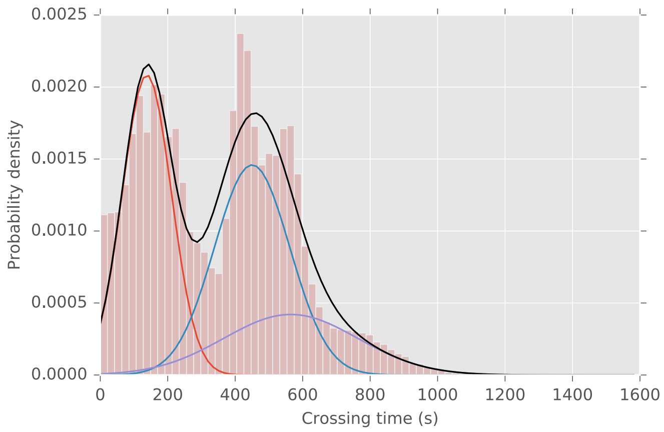

Fig. 6. EDYYB37EH crossing times fitted with a 3 components GMM

Once fitted, a GMM can work either as a *classifier* or a *predictor*: these two ways will be used hereafter. As a *classifier*, the model is given a time (planned or predicted) and returns the index of the most probable GMM component for this value:

$$\text{GMM}_{\text{class}}(t) \rightarrow i_{\text{max}}$$

When used as a *predictor*, the GMM gives, for an input value, the probabilities that it has been drawn from the different components:

$$\text{GMM}_{\text{pred}}(t) \rightarrow (p_k^t)_{k=1}^{n_{\text{GMM}}}.$$

The next section describes the general approach to use Gaussian Mixture Models to build probabilistic traffic models.

C. General Model Structure

For each elementary sector, a series of GMM is fitted to model the entry time distributions and the crossing times. An additional model is fitted for each (major) airport to model take-off delays (difference) between the planned off-block time and the actual one.

When the models are conditioned on parameters that can take many values (e.g. the departure airport), the value set is divided in two: the set of values that concerns a majority of elements in the dataset and the rest. For example, amongst the 601 different departure airports appearing in the traffic dataset for elementary sector EDYYB37EH, 20 of them are the origin of more than 50% of the flights. For each value in this first, majority set, a separate GMM is fitted that will be used as a predictor. A single GMM is fitted for the second set and used as classifier to determine the distribution parameters for the different values grouped by classes (e.g. ICAO region). See Section V for the details.

A full traffic model is so defined by:

- A set of parameters: the features of the flight that will determine the GMM to use (e.g. the departure airport, the day of the week...)
- For each parameter: the list of GMM to apply. This list may contain either a single model like for the elementary sector crossing times or multiple models with different models corresponding to different values of the parameter, like for the airport of departure.

D. Compatibility with the probabilistic trajectory model

The previous subsections have shown how empirical entry and crossing time distributions can be approximated by Gaussian Mixture Models. It is described now how GMM-approximated distributions can be used in the probabilistic trajectory model described in Section III-A where the entry time and exit time distributions are approximated by Gaussian distributions.

When a GMM is used as a classifier, it returns which of the Gaussians is the most probable for a given value. On the other hand, if a GMM is used as a predictor, it returns the probabilities of its different Gaussian components. In the first case, the parameters of the selected Gaussian can be directly used in the probabilistic trajectory model. In the second case, a separate trajectory can be produced with the parameters of the different components, each with the probability of the corresponding component.

If more than one parameter are used, the joint probability table of the predicted probabilities is built (the parameters are assumed independent). The size of the joint probability table is exponential in the number of parameters. This provides a strong justification to keep both the number of parameters and components in the models as small as possible.

Formally, let us assume a sector crossing (entry and exit times) for flight f modelled by q parameters. From the full model, the mixture models $\mathcal{M}_k$ for k ranging from 1 to q have been selected for the flight given the values of the different parameters. This set of GMM is made of two subsets: q_1 models ($\mathcal{M}_k^e$) affecting the delta entry time and q_2 models ($\mathcal{M}_k^l$) affecting the crossing time. Each $\mathcal{M}_k$ has n_k components of weights $w_{(k,l)}$ (with l from 1 to n_k). A GMM used as a classifier has a single component of weight 1.

The sector crossing will be represented by $n = \prod_{k=1}^{q} n_k$ probabilistic sector sequences and the set of possible GMM component combinations is the cross product of the integer sequences from 1 to n_k:

$$\mathcal{I}_f = \times_{k=1}^{q}\{1...n_k\}.$$

$\mathcal{I}_f$ is assumed to be ordered in some way (e.g. lexicographical order) so that $\mathcal{I}_{(f,i)} = (l_{(f,i,1)}, ..., l_{(f,i,q)})$ selects the i^{th} combination of GMM components. Then the probability of each of the n sector sequences is

$$p_{(f,i)} = \prod_{k=1}^{q} w_{(k,l_{(f,i,k)})}$$

and the entry and exit time distribution parameters are

$$\mu^e_{(f,i,j)} = \sum_{k=1}^{q_1} \mu_{(\mathcal{M}_k^e, l_{(f,i,k)})}$$

$$\sigma^e_{(f,i,j)} = \sqrt{\sum_{k=1}^{q_1} \sigma^2_{(\mathcal{M}_k^e, l_{(f,i,k)})}}$$

$$\mu^l_{(f,i,j)} = \mu^e_{(f,i,j)} + \sum_{k=1}^{q_2} \mu_{(\mathcal{M}^l_k, l_{(f,i,k)})}$$

$$\sigma^l_{(f,i,j)} = \sqrt{(\sigma^e_{(f,i,j)})^2 + \sum_{k=1}^{q_2} \sigma^2_{(\mathcal{M}^l_k, l_{(f,i,k)})}}$$

where j is the index of the elementary sector crossing in the i^{th} probabilistic trajectory.

V. Application

To fix the ideas, practical details on how entry and exit times of a single elementary sector can be modelled using the framework just described are given now. The model uses the airport of departure and the flight state (airborne or not) as mixture selection parameters. It is built for MUAC elementary sector EDYYB5KL. The historical dataset is built from the traffic of 3 consecutive AIRAC cycles (1607, 1608 and 1609). It contains 91389 crossings of EDYYB5KL. The model is used to derive occupancy count distributions for traffic predicted on the 5th of May, 2017.

A. Model building

The model describes off-block delays, delta entry times and crossing times by three series of GMM. Note that the off-block delays are airport specific, the delta entry times depend on both the sector of interest and the departure airport while crossing times depend on the elementary sector only.

1) Off-block delay and delta entry time models: Off-block delay and delta entry time models use the departure airport as parameter. Both model types are built the same way.

Traffic flying through a given sector potentially originates from a wide number of airports (552 in this case). Fitting separate GMM for each of these airports would be a large undertaking. To limit the effort, only the 11 most frequent airports (representing 59% of the traffic) have their off-block delay and delta entry time empirical distributions modelled by a GMM. A *residual* model is built for the remaining 541 airports.

The following table lists the 11 airports representing close to 60% of the traffic and the number of components of the fitted GMM:

Airport	%	n
EGGL	17.00	1
EGKK	12.19	3
EGSS	8.71	2
EGGW	4.50	2
LFPG	4.48	2
EDDF	2.71	2
EBBR	2.34	1
EGLC	2.30	2
EDDM	1.65	2
LEBL	1.58	1
EDDL	1.54	2

To build the *residual* model, a single GMM is fitted to the off-block delay or delta entry time distributions for all the remaining departure airports. In this case, the fitted GMM has 3 components. The remaining airports are then grouped by ICAO region (i.e. the two first letters of their ICAO code). The average off-block delay or delta entry time is then computed per region. This average is then input to the GMM used as a classifier to determine for each region the most probable component. The parameters (mean and standard deviation) of the selected component are used as the parameters of the off-block or delta entry time distribution for any flight originating from the region.

2) Crossing time model: A single 3 components GMM is fitted to model the sector crossing times. It has the following parameters:

i	w_i	μ_i (s)	σ_i (s)
1	.37	115.04	77.38
2	.31	330.67	159.85
3	.32	474.25	90.54

3) The complete set of models: In total the traffic model for sector EDYYB5KL is composed of 25 GMM fitted to historical data:

- 12 GMM for the off-block delay models: 11 predictor GMM for the most frequent airports and one additional classifier GMM used for all the remaining airports.
- 12 GMM for the delta entry time models: 11 predictor GMM for the most frequent airports and one additional classifier GMM used for the remaining airports.
- 1 predictor GMM to model sector crossing times.

B. Model use

The probabilistic traffic model is applied to the list of flights predicted for given target and look-ahead times. The list of flights known at a given time as well as their predicted trajectories is extracted from the Enhanced Traffic Flow Management System (ETFMS) logs for the day of interest.

The model selection parameters are extracted for each flight (in case of the model example given above the airport of departure and the state — airborne or not — of the flight). These parameters are used to select the models (set of GMM) that will be applied to compute the distribution parameters. If the off-block time is predicted (the flight is not yet airborne), the off-block delay model is used to compute the parameters of entry time distribution.

The currently predicted delta entry and crossing times of the flight are then input to the selected models and the returned probabilities are used to compute the parameters of the entry and exit time distributions for all the combinations of mixture components. So, for each flight, the application of the model results in a set of probabilistic trajectories.

The sets of probabilistic trajectories computed for all flights in the list are fed in the algorithm described in the section III-B to compute the occupancy count distributions $\Theta_{(s,t,)}$.

VI. Results

Figure 7 and Figure 8 show the occupancy count distributions, actual and predicted counts around respectively 11:00

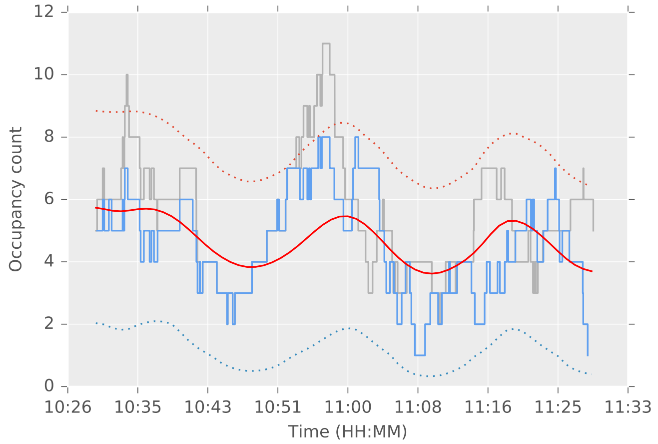

Fig. 7. Occupancy count distributions (red and dashed) along actual (blue) and baseline (grey) occupancies as predicted at 10:30 for 11:00

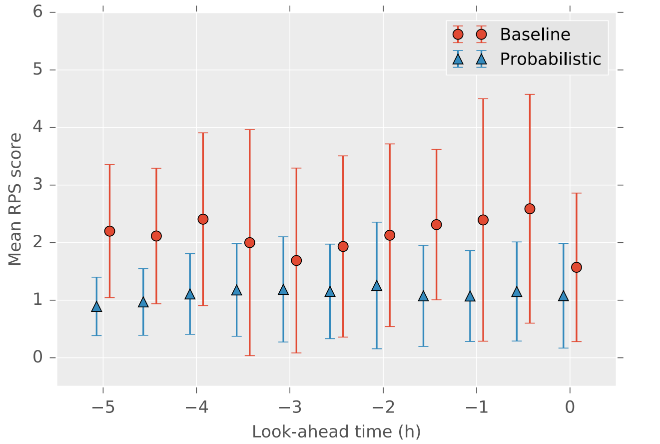

Fig. 9. RPS means and standard deviations for baseline (red) and probabilistic (blue) counts

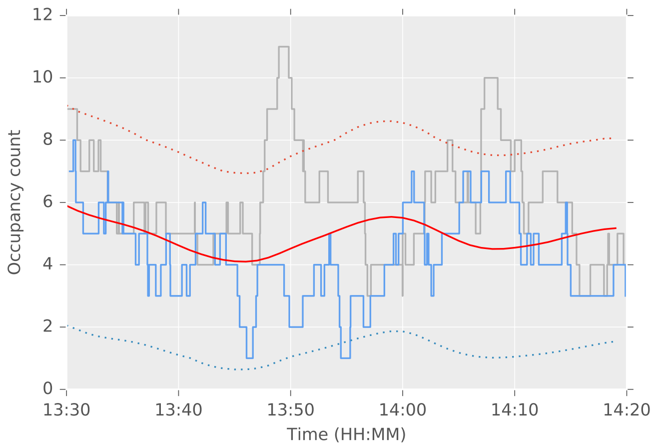

Fig. 8. Occupancy count distributions (red and dashed) along actual (blue) and baseline (grey) occupancies as predicted at 13:00 for 14:00

and 14:00 on the 5th of May 2017 for MUAC elementary sector EDYYB5KL using the approach described in this paper. The predictions were done 30 minutes and 1 hour in advance. The red curve represents the means of the occupancy count distributions computed every minutes. The (blue and red) dashed lines give the 90% interval computed from the occupancy count distributions (respectively the 5% and 95% quantiles). The blue curve gives the sector instantaneous occupancy counts as computed from the actual flight profiles extracted from the AllFT+ archive of the day. The grey curve is the occupancy counts as computed from the flights known at the time of prediction (baseline).

To further assess the model, probabilistic predictions (*Probabilistic counts*) and current occupancy count predictions (*Baseline counts*) were compared to the occupancy counts computed from the final profiles available in the corresponding AllFt+ archive (*Actuals counts*). The comparison was done for 37 target times falling every half an hour from 05:00 to 23:00 on the 5th of May, 2017. For each target time t, the predictions were compared for 11 look-ahead times (l) ranging, every half an hour, from $t - 5h$ to t.

The comparisons were done using the *Ranked Probability Score* (RPS) ([8], [9]).

If $F^{\Theta}_{(s,t,l)}$ is the cumulative distribution function of the occupancy count distribution $\Theta_{(s,t,l)}$ for sector s at time t predicted with look-ahead time l:

$$F^{\Theta}_{(s,t,l)}(n) = \sum_{i=0}^{n} \Theta_{(s,t,l)}(i)$$

and $H[n]$ is the discrete Heaviside step function:

$$H[n] = \begin{cases} 0, & n < 0, \\ 1, & n \geq 0, \end{cases}$$

then, for the probabilistic count prediction $\Theta_{(s,t,l)}$ and the actual count $o_{(s,t)}$, the RPS is computed as follows:

$$\text{RPS}\left(F^{\Theta}_{(s,t,l)}, o_{(s,t)}\right) = \sum_{n=0}^{\infty} \left(F^{\Theta}_{(s,t,l)}(n) - H[n - o_{(s,t)}]\right)^2$$

The RPS has the useful property of being able to handle both probabilistic predictions (the probabilistic counts) and deterministic predictions (the baseline counts): Deterministic predictions are considered as distributions with single value of probability 1. In this case the RPS is equal to the absolute error.

Figure 9 and Table I show the means and standard deviations of the scores computed between the actual counts and, respectively, the baseline counts (red) and the probabilistic counts (blue) for the different look-ahead times (lower scores mean better results).

Statistical tests applied to both the means and standard deviations show, at a 5% significance level, that:

- The baseline and probabilistic score standard deviations are significantly different for all the look-ahead times.

TABLE I
MEANS AND STANDARD DEVIATIONS OF THE BASELINE AND PROBABILISTIC SCORES

l	Baseline		Probabilistic	
(h)	mean	stdev	mean	stdev
-5.0	2.20	1.155	0.89	0.507
-4.5	2.12	1.177	0.97	0.579
-4.0	2.41	1.500	1.11	0.701
-3.5	2.00	1.963	1.18	0.804
-3.0	1.63	1.606	1.19	0.914
-2.5	1.93	1.574	1.15	0.821
-2.0	2.13	1.586	1.26	1.100
-1.5	2.31	1.306	1.08	0.878
-1.0	2.39	2.106	1.07	0.789
-0.5	2.59	1.987	1.15	0.861
0.0	1.57	1.290	1.08	0.910

The probabilistic prediction scores being less spread, there is a reduction in uncertainty on the count predictions which, according to the *Capacity buffer theory* [2], would lead to a capacity increase.

- The baseline and probabilistic means are significantly different for all look-ahead times except for predictions made at time t (0 hour look-ahead time) and at $t - 3h$: In the majority of the cases, the probabilistic count predictions are more accurate.

The stability over (look-ahead) time of the probabilistic prediction score has also to be noted as it would mean that probabilistic prediction provides more accurate and stable count predictions earlier in time.

VII. Conclusion and Further Research

Based on the probabilistic trajectory model and the algorithm presented in [1], the paper described a flexible and extensible approach based on historical data to attach uncertainty to traffic demand: The empirical take-off delay, delta entry time and crossing time distributions conditioned on selected parameters are modelled by Gaussian Mixture Models (GMM). The approach allows to model the multi-modal entry and crossing time distributions observed in ATM. When applied, the GMM-approximated distributions can be however represented as a combination of probable trajectories each having Gaussian entry and exit time distributions which are more tractable. These sets of trajectory combinations are input to a polynomial time algorithm computing occupancy count distributions.

The flexibility of the approach allows to select the set of most relevant model parameters and while being exponential in the number of parameters, it is shown that simple models (3 parameters modelled by GMM with up to 3 components) are already able to provide sensible results: Current results show not only a significant reduction in the spread of the prediction scores but also a general improvement of the quality of the predictions made using the probabilistic models.

The approach, limited here to the crossing of a single elementary sector, is being combined with probabilistic sector sequences necessary to model the uncertainty on the actual route flown by the aircraft.

Detailed performance analysis is on going. At this stage however, all computations related to the usage of the model always remained below one minute on standard desktop computers.

The work presented here opens several further research questions, amongst which:

- How to determine the most significant inputs or parameters to be taken into account when building traffic models?
- Is modelling time (or other) distributions in ATM using Gaussian mixture applicable/useful to other purposes than occupancy count distributions as described here?
- The use of Bayesian modelling should be explored when fitting the mixture models to the historical data. It would allow to inject some prior knowledge while determining the mixture parameters and avoid the tendency of BIC-based model determination to lead to models with a high number of components.

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