Fuel Saving
by Gradual Climb Procedure

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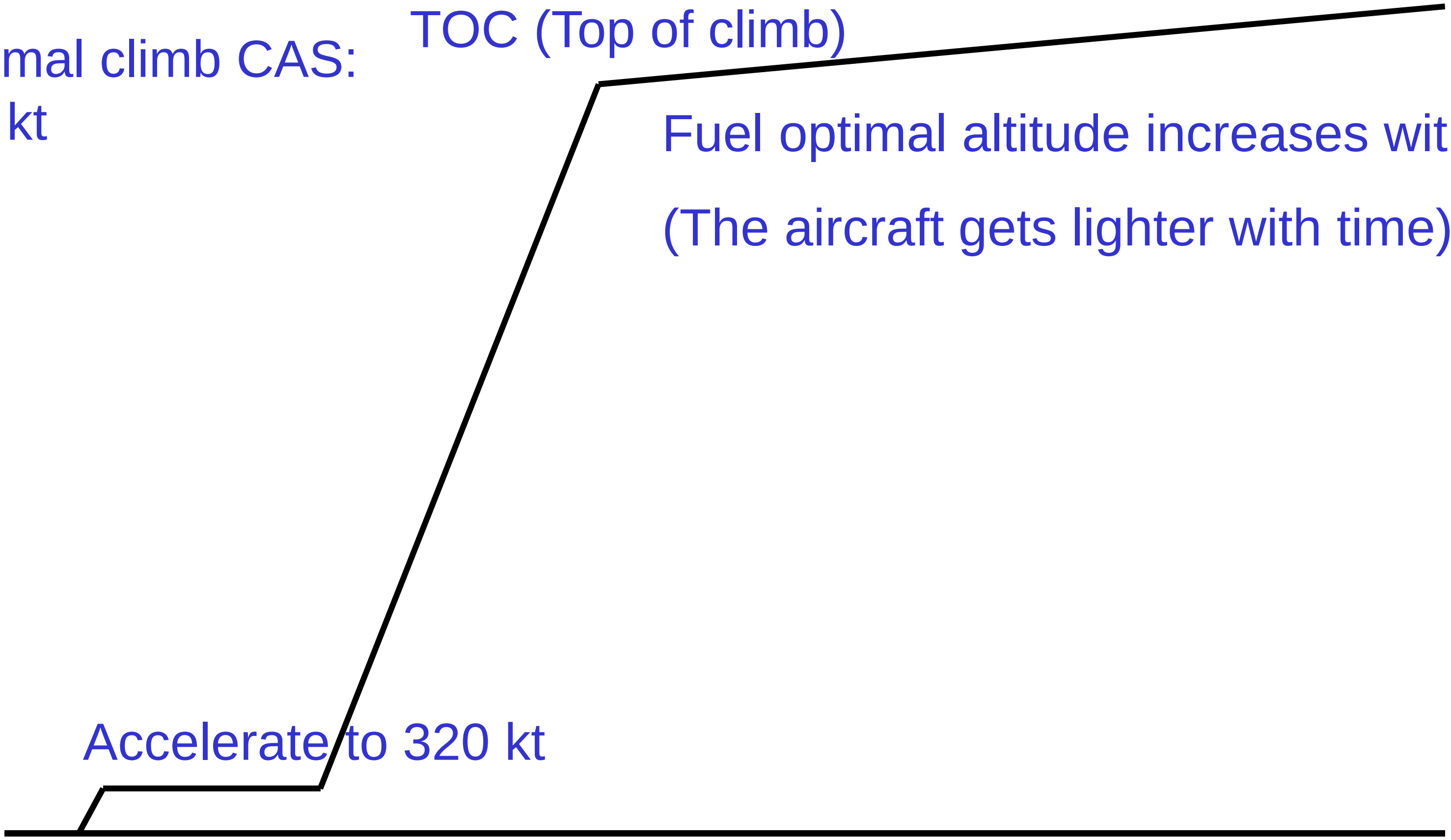
Single Flight Optimal Trajectory



Optimal climb CAS:
320 kt

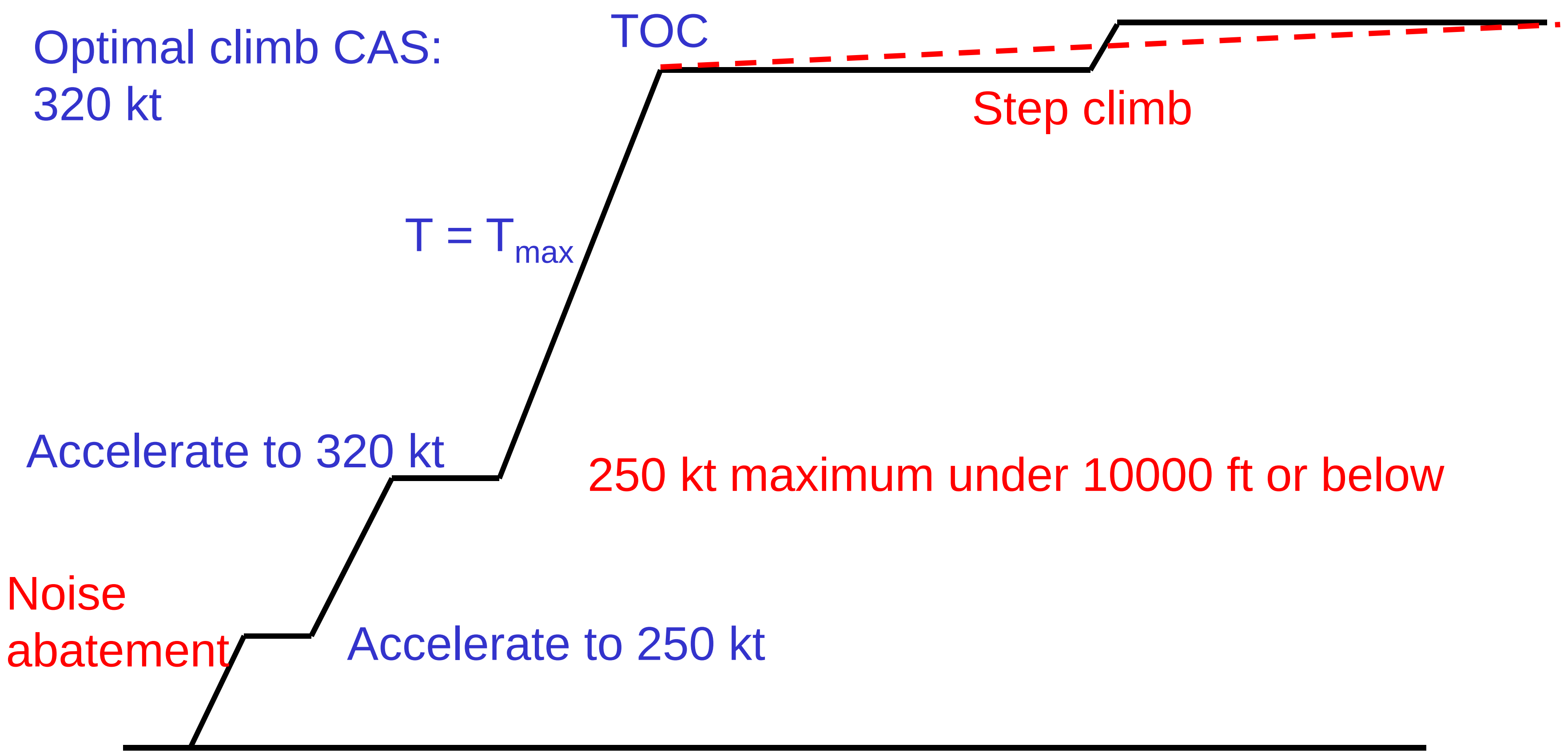
TOC (Top of climb)

Fuel optimal altitude increases with time

(The aircraft gets lighter with time)

Accelerate to 320 kt

Real World Optimal Trajectory



Optimal climb CAS: 320 kt

TOC

Step climb

$T = T_{max}$

Accelerate to 320 kt

250 kt maximum under 10000 ft or below

Noise abatement

Accelerate to 250 kt

- Basically, the higher altitude is better in terms of fuel consumption.

Fuel Consumption and Thrust

- Maximum Continuous Thrust (MCT) / Maximum Climb Thrust (MCL)… so-called maximum thrust
  - MCT is not the most fuel efficient, because the engine is designed to be the most fuel efficient during cruise.
  - Saving in climb thrust is achieved by reducing the rate of climb (ROC).
- The aircraft is more fuel efficient at higher altitude.
  - Small ROC means TOC moves further.

→Which impact is bigger?

1) “Fuel saving by lower thrust” or
2) “Fuel saving to reach TOC earlier with MCT”?

→Numerical optimization approach

Problem formulation (1)

Aircraft dynamics (no wind)

$$\dot{x} = v\cos\gamma$$
$$\dot{z} = v\sin\gamma$$
$$\dot{v} = \frac{T-D}{m} - g\sin\gamma$$
$$\dot{\gamma} = \frac{L}{mv} - \frac{g\cos\gamma}{v}$$
$$\dot{m} = f(M,T)$$
$$T = T_{ratio}(T_{\max} - T_{\min}) + T_{\min}$$
$$T_{ratio} = [0 \quad 1]$$

Control (optimization) variables

$$\mathbf{u} = [\dot{\gamma}, T_{ratio}]^T$$

- Point mass model
  - No wind considered
  - No lateral motion considered
- BADA 4 model
  - B777-300 (Engine: GE)
- Objective function to be minimized:

$$J = \frac{100}{3600} CI \cdot \underbrace{t_f}_{\text{Flight time}} + 0.453592 \underbrace{\int_0^{t_f} -\dot{m}dt}_{\text{Fuel consumption}}$$

  - Cost index :
    - 100 [100 lb/hour] (= 2.78 lb / second)
  - Unit: [lb]

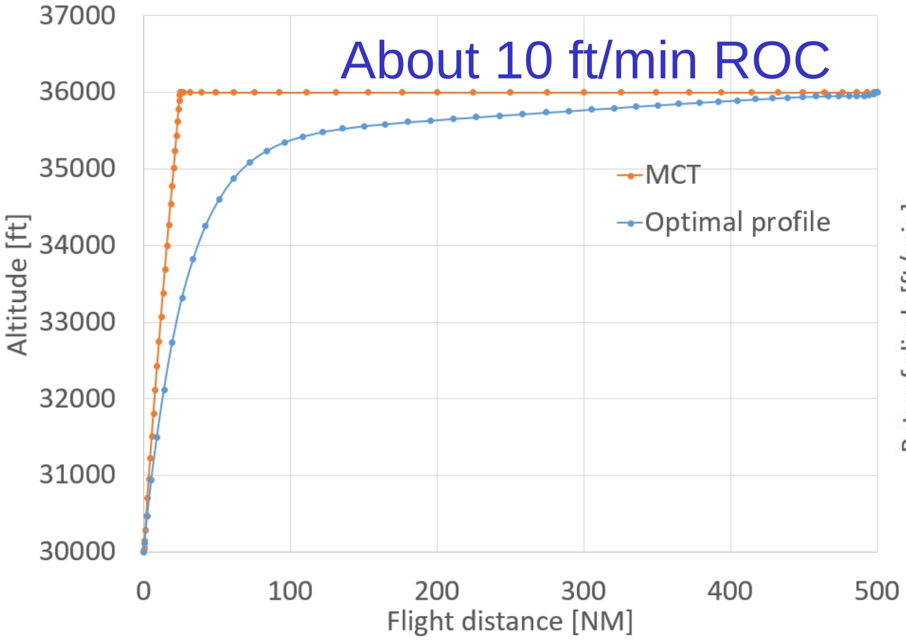
Problem formulation (2)

	Value
Initial altitude	30000 ft
Terminal altitude	36000 ft
Initial/Terminal Mach	0.83
Initial climb angle	1.0 deg
Flight distance	500 NM
Initial Weight	540,000 lb

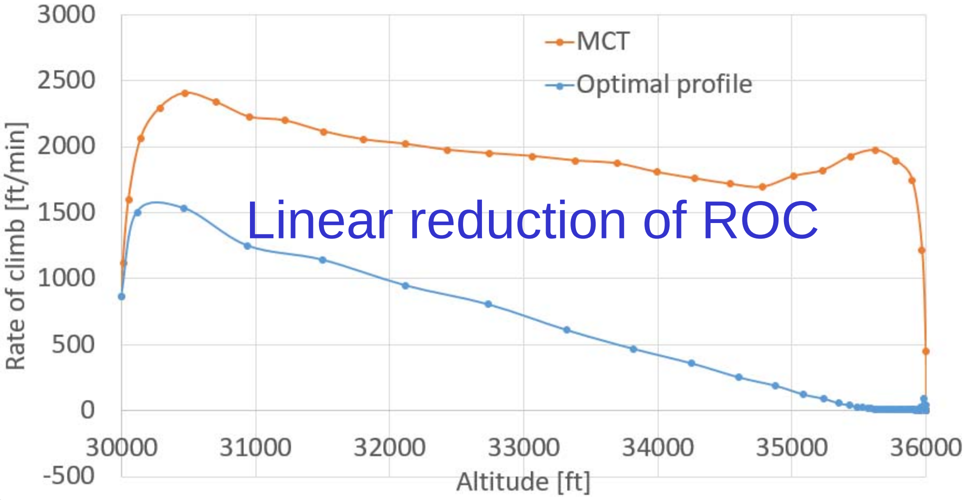
- Optimal climb: single-stage optimal control problem
- MCT climb: 2-stage optimal control problem

  ➔ NLP (Nonlinear programming) solver is used.

Optimal Trajectory and Current MCT Trajectory



About 10 ft/min ROC



Linear reduction of ROC

	MCT	Optimal climb	Difference
J (Objective function) [lb]	27,160	27,095	65
Fuel consumption [lb]	16,700	16,653	47
Flight time [s]	4,744	4,736	8

Optimal climb profile cannot be implemented in the current FMC and ATC. ➔ Sub-optimal practical climb profile is proposed.

Purpose of This Research

- To propose a new climb profile which saves fuel compared to the MCT climb.
  - The new profile should be possible within the current FMC.
  - Additional pilot tasks should be minimized.
  - Negative impact to ATC should be minimized.
  - Potential fuel savings should not be negligible.
    - The cumulative effect is also important because most aircraft are expected to change the climb profile.

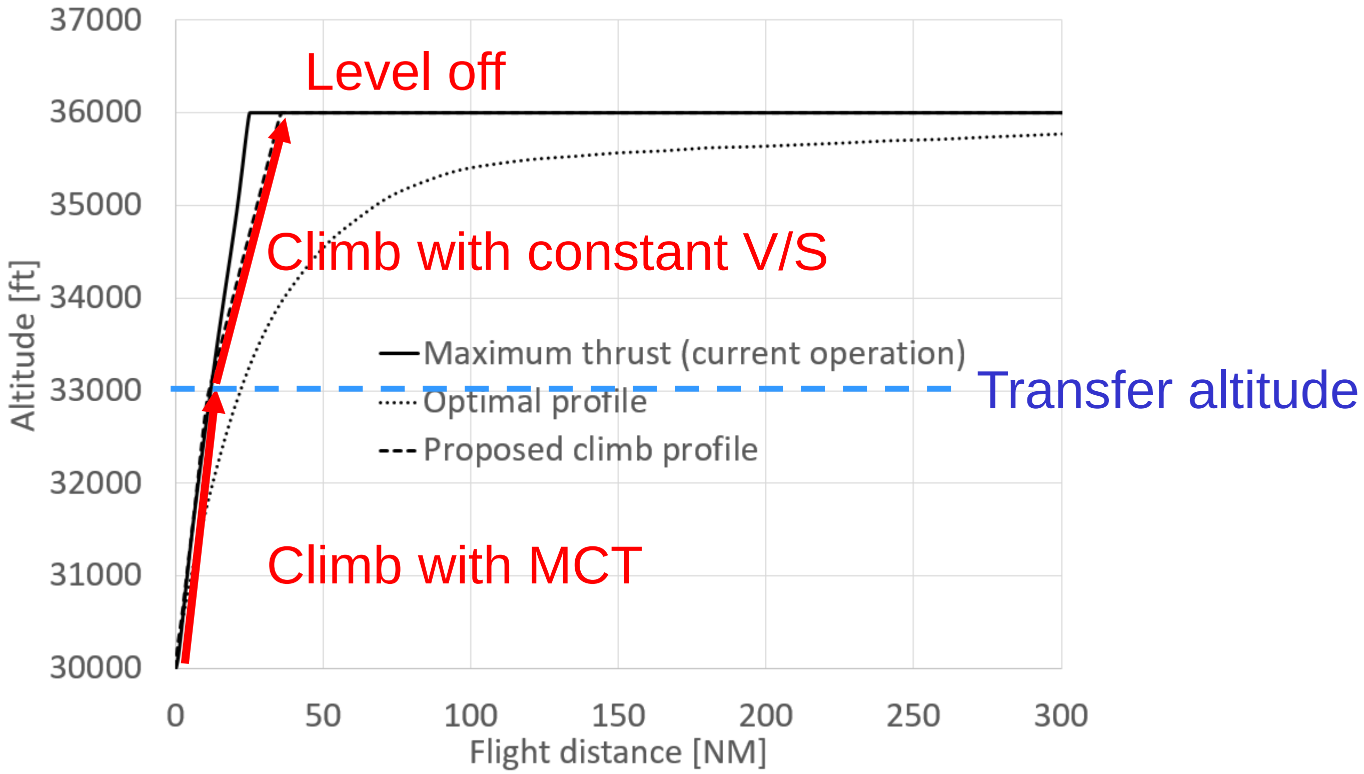
Current FMC Climb

- Two basic FMC modes to climb
  - VNAV SPD is usually applied during climb.
  - V/S (vertical speed) mode requires a target vertical speed
    - Climb trajectory can be changed.

    ➔ V/S mode is used here.

	VNAV SPD mode	V/S mode
Pitch control	Track target speed	Track target V/S
Thrust control	MCT	Track target speed
Target speed	Calculated by FMC	Calculated by FMC
Note	V/S is uniquely determined from the thrust.	Target V/S is set manually.

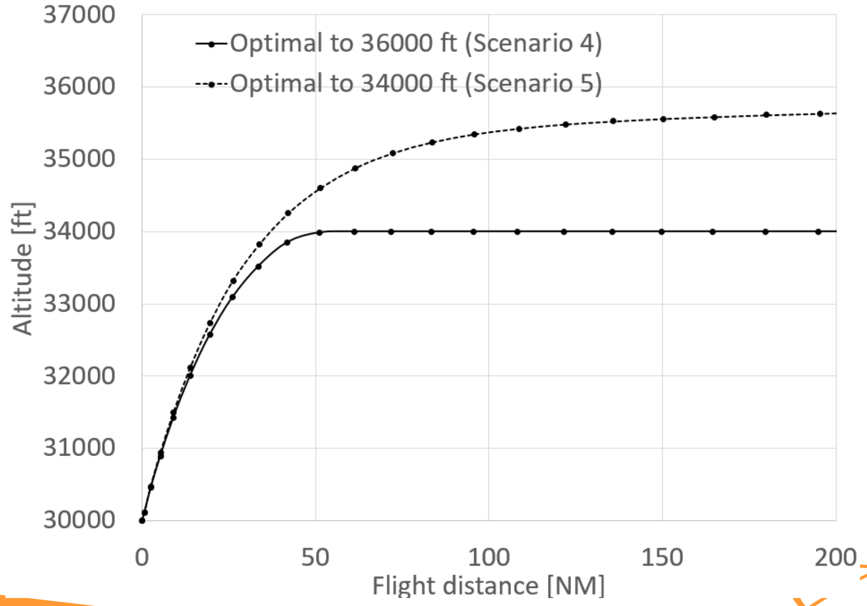
Proposed Climb Profile



- Climb with MCT to “transfer altitude”.
- Climb with constant V/S to cruise altitude.

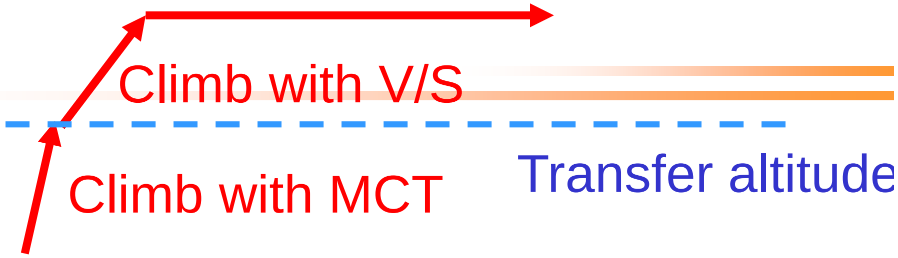
Calculation Conditions

- B777-300
  - About 2000 ft/min ROC with MCT
- Two scenarios
  - Climb to optimal altitude (36000 ft@540000 lb)
  - Climb to lower altitude (34000 ft) due to ATC instruction
- Target V/S
  - 500 ft/min or 1000 ft/min
- Transfer altitude
  - 30000 ft - 34000 ft
- 3-stage optimization
  - MCT climb
  - Constant V/S climb
  - Cruise



Optimal profiles under two scenarios

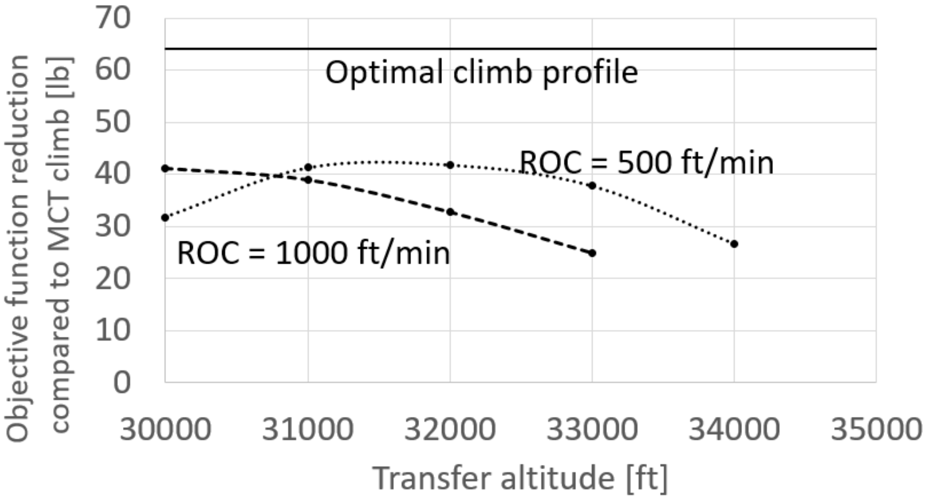
Fuel Saving by Proposed Procedure



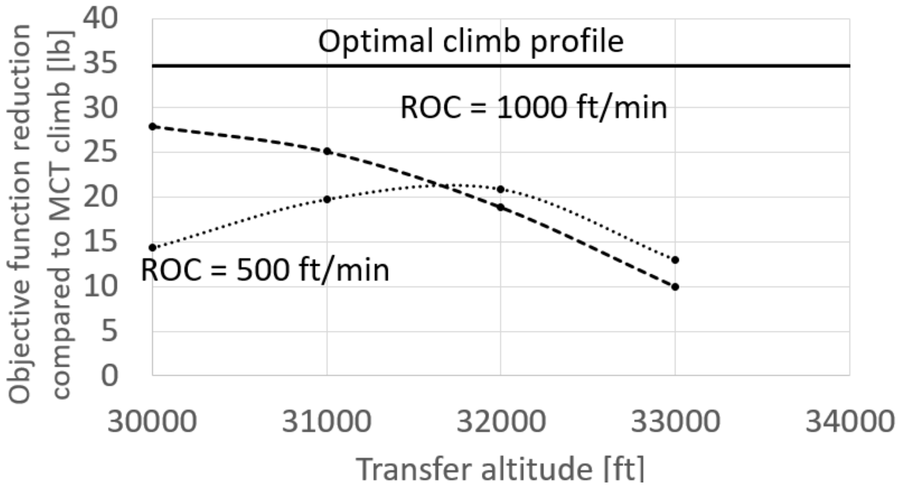
Climb with V/S

Transfer altitude

Climb with MCT



Climb to 36000 ft



Climb to 34000 ft

- Proposed procedure saves 30-40 lb fuel if appropriate transfer altitude and V/S are chosen.
  - TOC moves 1-6 min/10-50 NM forward.

Impacts to Pilots & ATC

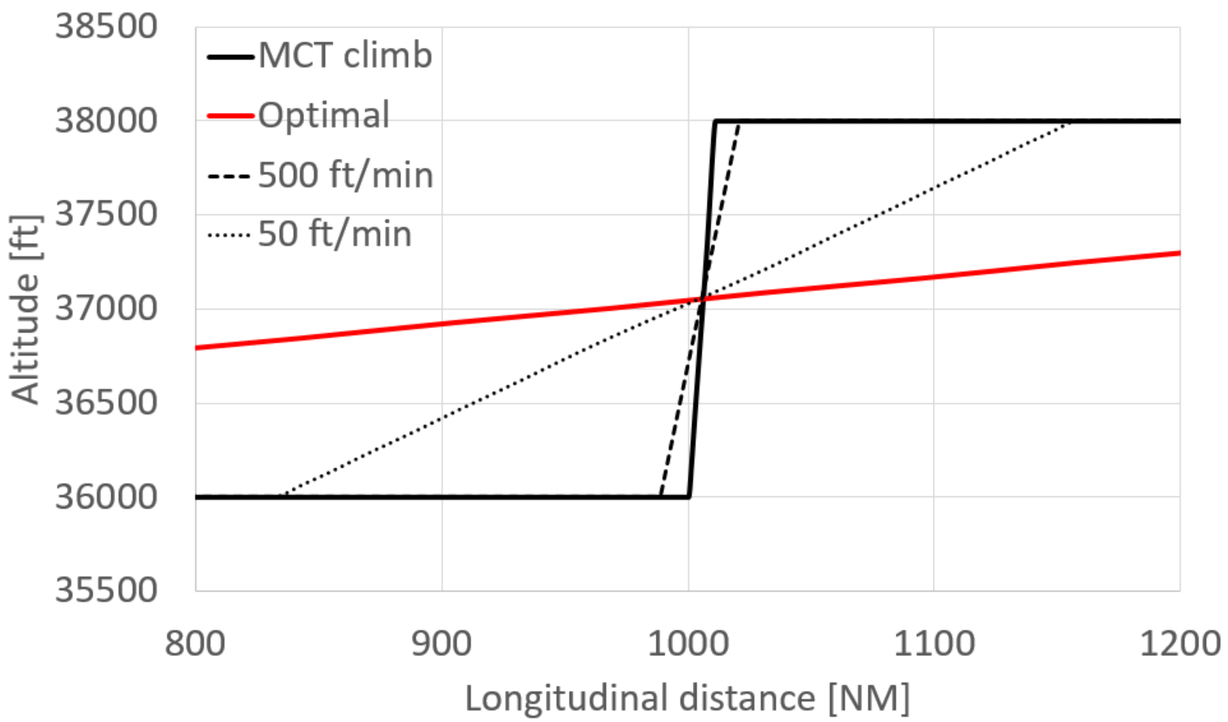
- Impacts to pilots
  - Some additional tasks are needed.
  - Minimum 500 ft/min ROC is recommended for situational awareness in TCAS monitor.
- Impacts to ATC
  - No negative effect will be observed unless there is other traffic.
    - ATC does not instruct V/S or time limit of climb.
  - 1-6 minutes delay to reach TOC might be an issue when there is other traffic nearby.
  - 500 ft/min climb is not slow.
    - V/S with MCT near TOC is less than 1000 ft/min for most aircraft types.

VNAV PATH
- Push VNAV button.

VNAV SPD

V/S
- Push V/S button.
- Select target V/S.

Low ROC Climb in Step Climb



- During step climb, MCT is usually applied.
  - Low ROC with V/S mode will save fuel.
- 3-stage optimization (cruise-step-cruise)
  - Appropriate constraints are set in “step” stage.

Calculation Results for Step Climb

Scenario	**1**	**2**	**3**
Aircraft type	B777-300	B777-300	A330-300
Initial weight [lb]	540,000	610,000	440,000
Initial altitude [ft]	36,000	33,000	37,000
Terminal altitude [ft]	38,000	35,000	39,000
Initial/Terminal Mach	0.83	0.83	0.80
Cost index	100	100	100
Flight distance [NM]	2,000	2,000	2,000
Objective function [lb] (Compared to MCT climb)			
Optimal climb	–88	–93	–71
50 ft/min climb	–55	–57	–41
500 ft/min climb	–28	–29	–14
1000 ft/min climb	–14	–17	–1
MCT climb	0	0	0

Conclusions

- A new practical climb procedure (gradual climb procedure) is proposed.
  - Gradual climb procedure is applicable in the current FMC using V/S mode.
  - 30-40 lb fuel saving per climb is expected with B777-300.
    - Cumulative effect will be significant because most departure aircraft can apply this procedure.
  - Negative effects to pilots and ATC are limited.
    - Pilots have to perform some additional tasks.
  - The similar procedure can be applied in step climb procedure.
    - Step climb is operated by long-haul flights only.
- Detailed conditions (appropriate V/S & transfer altitude with temperature, wind, aircraft type, etc) will be further investigated.

Fuel Saving by Gradual Climb Procedure

Thank you for your attention!

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Scenario 1
(Climb to 36000ft)

ROC [fpm]	Transfer altitude [ft]	J [lb]	Fuel [lb]	Flight time [s]
MCT	-	27160	16700	3765
Optimal	-	27095	16653	3759
500	33000	27119	16663	3764
1000	30000	27118	16666	3763