Probabilistic Aircraft Conflict Detection Considering Ensemble Weather Forecast

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Outline

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Motivation

A **promising approach** to improve current prediction and optimisation mechanisms towards meeting Single European Sky goals is the **modelling, analysis, and management of the uncertainty present in ATM**.

$\Downarrow$

One of the main sources of uncertainty that affect the ATM system, as identified by the **ComplexWorld Research Network,** is **weather uncertainty.**

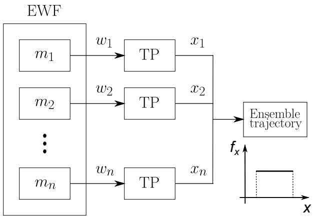
$\Downarrow$

The **objective of this work** is to analyze the effects of **wind uncertainty on conflict detection.**

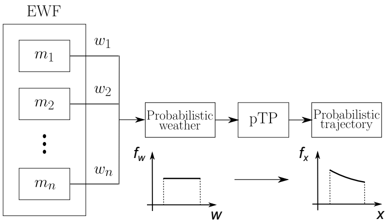
Approach I

- **Wind uncertainty:** provided by **Ensemble Weather Forecasts**; typically, a collection of 10 to 50 forecasts.

  According to the IMET project, there are **two approaches** for trajectory prediction:



Ensemble trajectory prediction



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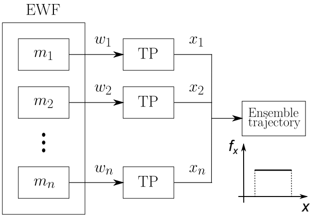

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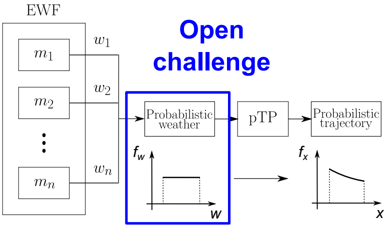
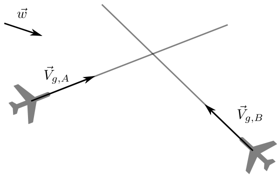


Probabilistic trajectory prediction

Approach II

- **Conflict detection:** between two aircraft, both flying at constant airspeed, course and altitude.



The conflict is characterized by:

  - **minimum distance** (d_{min}),
  - **time to minimum distance** ($t_{d_{min}}$),
  - **probability of conflict** (P_{con}).

- **Analysis**: based on the **Transformation of Random Variables**, the wind probability density functions (PDFs) are evolved to obtain the PDFs and values of the conflict indicators.

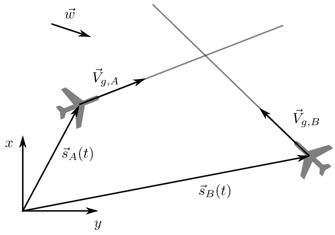
Applicability

- Air-traffic **safety and efficiency** may benefit from the inclusion of weather uncertainty in automated conflict detection at all levels:
  - **Long range** (time horizon of several hours)
    Trajectories may be **strategically deconflicted** even prior to the take-off.
  - **Mid** (tens of minutes) and **short range** (seconds to minutes)
    Decision support tools and **safety nets** (e.g. MTCD and STCA) may notify the conflicts to the ATCOs according to their probability of occurrence, thus reducing the number of missed and false alerts.

Outline

Assumptions

- A **North-East reference system** is used.
- Both aircraft, A and B, are affected by the **same wind** ($\vec{w}$).
- The **wind components** (w_x and w_y) are **uncertain and statistically independent**.
- Both aircraft fly at **constant airspeed** (V_A and V_B), **constant course** (ψ_A and ψ_B), and the same **constant altitude**.



- The **initial positions** ($\vec{s}_{0,A}$ and $\vec{s}_{0,B}$), **airspeeds** and **courses** are **perfectly known**.
- The **initial separation** between aircraft is **greater than a separation requirement** D (e.g., 5 NM) and **they are approaching**.

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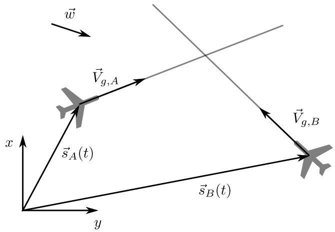

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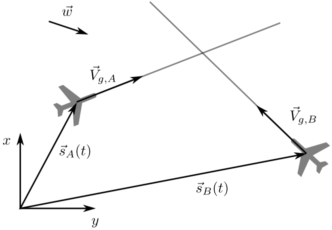

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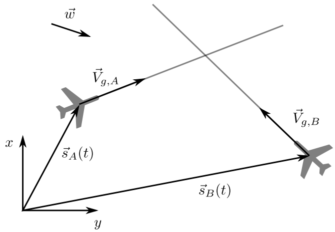

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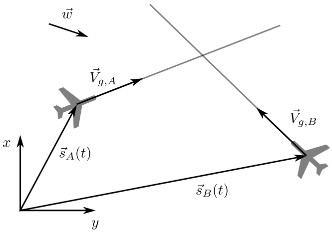

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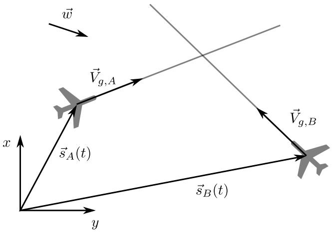

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Absolute motion of each aircraft

- **Position of each aircraft** at any time t:

$$\vec{s}_A(t) = \vec{s}_{0,A} + \vec{V}_{g,A} t,$$
$$\vec{s}_B(t) = \vec{s}_{0,B} + \vec{V}_{g,B} t.$$

- **Groundspeeds** $\vec{V}_{g,A}$ and $\vec{V}_{g,B}$ are obtained from the wind triangles:

$$\vec{V}_{g,A} = \vec{V}_A + \vec{w} = V_A \begin{bmatrix} \cos\left(\psi_A - \arcsin\left(w_{c,A}/V_A\right)\right) \\ \sin\left(\psi_A - \arcsin\left(w_{c,A}/V_A\right)\right) \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \end{bmatrix},$$
$$\vec{V}_{g,B} = \vec{V}_B + \vec{w} = V_B \begin{bmatrix} \cos\left(\psi_B - \arcsin\left(w_{c,B}/V_B\right)\right) \\ \sin\left(\psi_B - \arcsin\left(w_{c,B}/V_B\right)\right) \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \end{bmatrix}.$$

Crosswinds $w_{c,A}$ and $w_{c,B}$ are positive if they are from the left wing:

$$w_{c,A} = w_y \cos\psi_A - w_x \sin\psi_A, \quad w_{c,B} = w_y \cos\psi_B - w_x \sin\psi_B.$$

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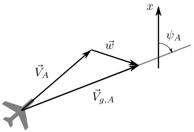

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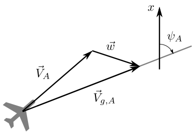

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The courses are known, but **the magnitudes of the groundspeeds are uncertain**

Relative motion between the aircraft

- The **conflict indicators** of this work only depend on the **relative motion** between the two aircraft.
- **Relative position** between the two aircraft at any time t:

$$\vec{s}(t) = \vec{s}_B(t) - \vec{s}_A(t) = \vec{s}_0 + \vec{V}_g t,$$

where

$$\vec{s}_0 = \vec{s}_{0,B} - \vec{s}_{0,A},$$
$$\vec{V}_g = \vec{V}_{g,B} - \vec{V}_{g,A}.$$

and the **relative groundspeed** $\vec{V}_g$ is given by:

$$\vec{V}_g = V_B \begin{bmatrix} \cos\left(\psi_B - \arcsin\left(w_{c,B}/V_B\right)\right) \\ \sin\left(\psi_B - \arcsin\left(w_{c,B}/V_B\right)\right) \end{bmatrix} - V_A \begin{bmatrix} \cos\left(\psi_A - \arcsin\left(w_{c,A}/V_A\right)\right) \\ \sin\left(\psi_A - \arcsin\left(w_{c,A}/V_A\right)\right) \end{bmatrix}.$$

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The magnitude and direction of the relative groundspeed are uncertain and, **under the hypotheses of this work, only affected by the crosswinds.**

Conflict indicators

The distance between the two aircraft at any time, $d(t)$, is the magnitude of the relative position:

$$d(t) = \|\vec{s}(t)\| = \sqrt{s_0^2 + 2\vec{s}_0\vec{V}_g t + V_g^2 t^2}.$$

- **Minimum distance:**

$$d_{min} = \sqrt{s_0^2 - \left(\vec{s}_0\vec{V}_g\right)^2 / V_g^2}.$$

- **Time to minimum distance:**

$$t_{d_{min}} = -\left(\vec{s}_0\vec{V}_g\right) / V_g^2.$$

- **Probability of conflict:**

$$P_{con} = P[d_{min} \leq D].$$

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$$P_{con} = P[d_{min} \leq D].$$

These conflict indicators **depend on the crosswinds** through $\vec{V}_g$ **but not on the along track-winds**.

Tranformation of Random Variables I

Basis of the **bivariate transformation**:

- w_x and w_y are random variables with joint PDF $f_{w_x,w_y}(w_x, w_y)$, and R is the set in the $w_x w_y$-plane where $f_{w_x,w_y}(w_x, w_y) > 0$.
- v_1 and v_2 are two random variables whose PDFs are to be found, and $v_1 = g_1(w_x, w_y)$ and $v_2 = g_2(w_x, w_y)$ define a one-to-one transformation of R onto a set S in the $v_1 v_2$-plane.
- If $w_x = h_1(v_1, v_2)$ and $w_y = h_2(v_1, v_2)$, then the PDF of v_1 and v_2 is given by

$$f_{v_1,v_2}(v_1, v_2) = \begin{cases} f_{w_x,w_y}(h_1(v_1, v_2), h_2(v_1, v_2))\,|J| & \text{if } (v_1, v_2) \in S, \\ 0 & \text{otherwise,} \end{cases}$$

where $|J|$ is the absolute value of the Jacobian determinant

$$J = \begin{vmatrix} \dfrac{\partial h_1(v_1, v_2)}{\partial v_1} & \dfrac{\partial h_1(v_1, v_2)}{\partial v_2} \\ \dfrac{\partial h_2(v_1, v_2)}{\partial v_1} & \dfrac{\partial h_2(v_1, v_2)}{\partial v_2} \end{vmatrix}.$$

Tranformation of Random Variables II

- **In this work, v_1 is any of the indicators** d_{min} or $t_{d_{min}}$, and v_2 is a dummy variable which has been chosen to be w_y.
- The PDF of v_1 can be obtained by integrating f_{v_1,v_2} in v_2

$$f_{v_1}(v_1) = \int_{-\infty}^{\infty} f_{v_1,v_2}(v_1,v_2)\mathrm{d}v_2$$

- The expected value, the typical deviation and the probability of v_1 being smaller than a given value are given by

$$E[v_1] = \int_{-\infty}^{\infty} v_1 f_{v_1}(v_1)\mathrm{d}v_1,$$

$$\sigma[v_1] = \left[\int_{-\infty}^{\infty} v_1^2 f_{v_1}(v_1)\mathrm{d}v_1 - (E[v_1])^2\right]^{1/2},$$

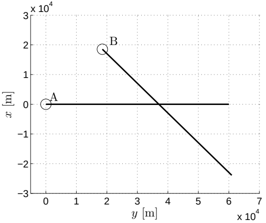
$$P[v_1 < a] = \int_{-\infty}^{a} f_{v_1}(v_1)\mathrm{d}v_1.$$

Outline

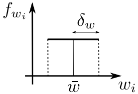
Scenario

- **Aircraft positions and speeds**

  $\vec{s}_{0,A} = [0,0]$, $\vec{s}_{0,B} = [18520, 18520]$ m,
  $V_A = V_B = 240$ m/s,
  $\psi_A = 90$ deg, $\psi_B = 135$ deg,
  $D = 9260$ m (5 NM).



- **Wind**
  Same **uniform probability distribution** for w_x and w_y



$$f_{w_i}(w_i) = \begin{cases} 1/(2\delta_w) & w_i \in [\bar{w}-\delta_w, \bar{w}+\delta_w], \\ 0 & \text{otherwise}, \end{cases}$$

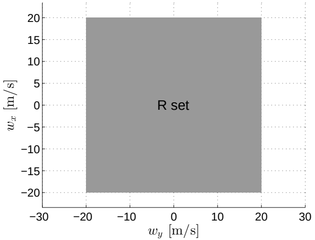
  where $i \in \{x, y\}$, $\bar{w} \in [-20, 20]$ m/s, and $\delta_w \in [0, 25]$ m/s.
  If $\bar{w} > 0$ the wind points Northeast and if $\bar{w} < 0$ it points Southwest (on average).
  Since w_x and w_y are statistically independent, the **joint PDF** is

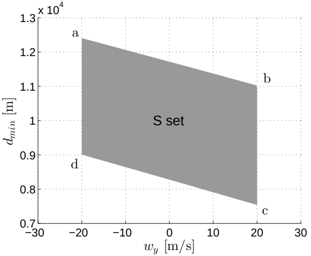
$$f_{w_x,w_y}(w_x, w_y) = f_{w_x}(w_x) f_{w_y}(w_y)$$

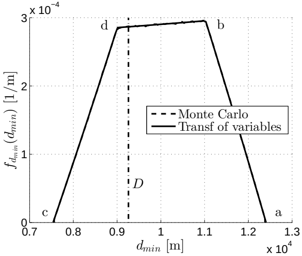
- Results are **validated by the Monte Carlo method** (8.4 million samples).

Minimum distance I

- Results for $\bar{w} = 0$ and $\delta_w = 20$ m/s:



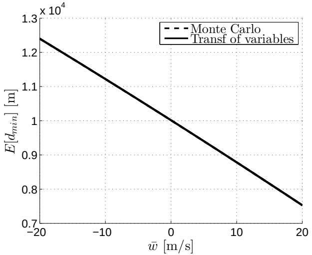




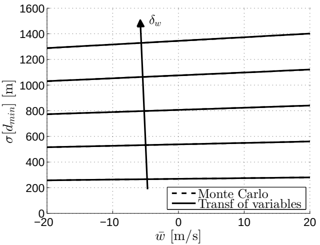
$E[d_{min}] = 10012$ m
$\sigma[d_{min}] = 1076$ m
$P_{con} = 28.0$ %

Minimum distance II

- Results for $\bar{w} \in [-20, 20]$ and $\delta_w = 5, 10, 15, 20, 25$ m/s:



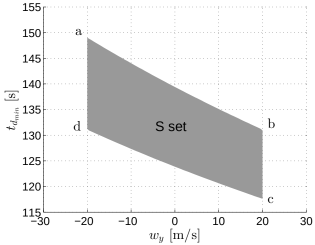
$E[d_{min}]$ does not depend on δ_w.
In this scenario, it decreases as $\bar{w}$ increases because the wind changes from pointing Southwest to pointing Northwest.

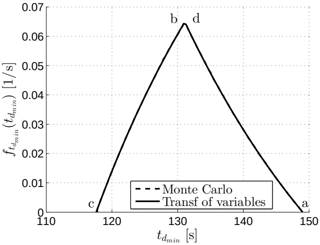


$\sigma[d_{min}]$ increases as δ_w increases.
Its dependence with $\bar{w}$ is very weak.

Time to minimum distance

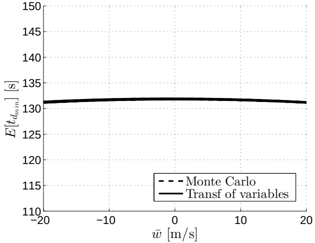
- Results for $\bar{w} = 0$ and $\delta_w = 20$ m/s:

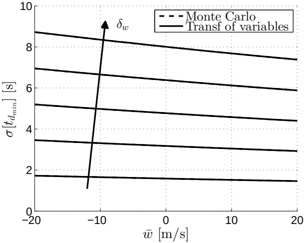




$E[t_{d_{min}}] = 131.9$ s
$\sigma[t_{d_{min}}] = 6.4$ s

- Results for $\bar{w} \in [-20, 20]$ and $\delta_w = 5, 10, 15, 20, 25$ m/s:

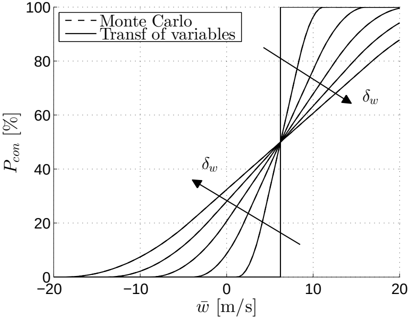




$E[t_{d_{min}}]$ is almost independent of $\bar{w}$ and δ_w.
$\sigma[t_{d_{min}}]$ increases as δ_w increases, its dependence with $\bar{w}$ is very weak.

Probability of conflict

- Results for $\bar{w} \in [-20, 20]$ and $\delta_w = 0, 5, 10, 15, 20, 25$ m/s:



The certainty that a conflict does exist or does not exist decreases as the wind uncertainty increases.

Outline

Conclusions

- A first step on the assessment of the **effects of wind uncertainty on conflict detection** has been presented.
- The presented approach is capable of taking as input **any type of wind distribution derived from Ensemble Weather Forecasts.** The determination of the PDF from the ensemble forecast is an **open challenge**.
- Under the hypotheses that the aircraft follow a constant course and they are affected by the same wind, it has been found that **the three considered indicators depend on the crosswinds seen by both aircraft, but not on the along-track winds.**
- **Numerical results** have been presented for a **particular scenario** to show the **potentiality of the proposed methodology**.

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Future Work

- Application to **other conflict indicators**.
- Obtention of wind distributions from **actual ensemble forecasts**.
- Formulation of the problem for trajectories composed of **segments with different courses**.
- Aircraft affected by a different wind obtained from a **statistically-correlated wind-field**.

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Thanks! Questions?