



references therein. In Ref. [7] the authors used a weighted degree distribution for light jet operations to understand the capability of airports to attract the use of Very Light Jets (VLJs). In Ref. [8] network theory tools were used to explore how they can assist in the understanding, monitoring and management of the performance of ATM systems with respect to safety issues (STCA signals). In Ref. [9], [10] the regional network of Lufthansa flights has been considered in order to investigate whether the strategies of that airline and its partners can be revealed by a network theory analysis. The objective of the work reported in Ref. [11] is to determine if network theory parameters can be utilized to identify unconnected city pairs that are most likely to connect in the future. In Ref. [12] the connectivity of the airport networks in China, Europe and US is investigated in the aim of understanding which network is most beneficial to final passengers in terms of travel time and which of the network features lead to such a result.

The papers mentioned above give just examples of how disparate are the topics where Network Theory tools can be successfully applied. In the present paper we start to investigate the issue of air transportation system with an emphasis on flights delay. Specifically we will present a preliminary analysis aiming at understanding how the airport network topology is related to the statistical properties of flights delay and how network topology affects the propagation of flight delays from an airport to another.

### III. DATABASE DESCRIPTION

The data used in the present investigation are provided by Eurocontrol Central Flow Management Unit (CFMU) under special confidentiality bounds. The provided data are the SegOut6 (*so6*) and the EXPand/Traffic demand file (*exp2*) files [13]. The period of investigation is the week June 1-7, 2011.

The *so6* files give a 4D description of trajectories. This is the basic file format used for queries, airspace load and other key SAAM (System for traffic Assignment & Analysis at Macroscopic level) functions. It stores SAAM 4D trajectories and limited flight information. These data contain points information about latitude, longitude, flight level, date and time of all segments of each flight recorded by the ATM units of the EUROCONTROL (ECTL) countries.

We processed two types of *so6* files: files containing data relative to the last filled flight plan (*m1*) and files containing data relative to updating of the the last filled flight plan with radar data (*m3*). The estimates derived from received flight plans are thus updated during the flight execution with radar tracks forwarded to the CFMU by the different ATC centres. Many factors can contribute to deviations between planned and executed trajectories.

The *exp2* files store flights information, but no trajectories. They are used for SAAM as a traffic demand for the assignment. Examples of flight information stored are the callsign and the type of flight, e.g. whether it was a General Aviation flight (G), a Military flight (M), a Non scheduled flight (N) or a Scheduled flight (S). From the callsign it is possible to get information about the airline that operated the flight.

In the preliminary investigation presented in this paper we use only part of the provided information. Specifically, we decide to not use the information about segments and to investigate the issue of delays between the last filled flight plan and the effective flight at the level of a single flight [14]. For each flight we consider as a proxy of the flight delay the difference between the arrival time reported in the *m3* file and the arrival time reported in the *m1* file. It is worth mentioning that the *exp2* file contains a column called “ATFM DELAY” standing for *Numeric Air Traffic Flow Management Delay*. This is the difference between the take off time (CTOT) calculated by Civil Aviation Safety Authority (CASA) and the estimated take off time (ETOT). In a future extension of this work, we plan to compare the result obtained by using the above defined proxy of delay, with the one obtained by considering the “ATFM DELAY”.

### IV. AIR TRAFFIC NETWORKS: DEFINITION

We apply methods of complex network theory to ATM. It is possible to define many different types of networks of the air traffic, depending on the research question one is interested in. Since one of our objectives is to study the dynamics of delay in the European airspace, we will consider the airport network. As we will see below, it is possible to define at least two types of airport networks.

The first type of networks considers the airports as nodes. Two airports are connected by a link when at least one flight is present between the two airports. The number of flights in a given time interval between two airports gives the weight of the link (see also section V) between the corresponding nodes. More precisely, if in the considered time period there are  $n_{AB}$  flights which go from airport  $A$  to airport  $B$  and  $n_{BA}$  flights which go from airport  $B$  to airport  $A$ , we consider that nodes  $A$  and  $B$  in the network are connected by two links, one from  $A$  to  $B$  with weight  $n_{AB}$  and one from  $B$  to  $A$  with weight  $n_{BA}$ . This airport network, termed *flight network*, is therefore weighted and directed.

The empirical analysis of the real airport network shows that one can, in first approximation, consider a simpler version of the network. In fact, when one considers a time interval of a day (or longer), one observes that for most links it is  $n_{AB} \simeq n_{BA}$ , i.e. two airports are typically connected by the same number of flights in both directions. In order to test this approximation, we consider a specific day, namely June 1, 2011 (but similar results hold also for other days). For each pair of airports  $A$  and  $B$ , which are connected by at least one flight, we count  $n_{AB}$  and  $n_{BA}$ . In the left panel of Fig. 1 we show  $n_{AB}$  versus  $n_{BA}$ . Apart from some scattering due to small values of the number of flights observed close to the origin of the figure panel, we generically observe that the points lie quite close to the line  $n_{AB} = n_{BA}$ , as assumed above. Note that the same type of symmetry is observed if one plots the total number of flights departing from one airport versus the total number of flights landing in the same airport (see right panel of Fig. 1). Due to this approximate symmetry, in the following of this paper we will consider an undirected

network where nodes are airports and links are weighted with the total number  $w_{AB} \equiv n_{AB} + n_{BA}$  of flights connecting the two airports. We call this undirected and weighted network *route network*.

Given this distinction between the two networks, it is also important to consider a partition of the nodes and of the links of both networks due to the specific structure of our database. The database is provided by ECTL and includes flights that take off or land in an ECTL country or flights between two non-ECTL countries but that use the airspace of one or more ECTL countries. Therefore we consider two types of nodes, namely those associated to an airport in an ECTL country and those outside. In the investigated week the fraction of airports in an ECTL country ranges between 67% and 72%.

Similarly, we classify three types of flights. The majority of flights (between 74% to 80%) are between ECTL airports. A small fraction of flights, ranging between 2.2% and 2.7% is between two non ECTL airports. The remaining fraction of flights is between an ECTL airport and a non ECTL one, approximately equally split between incoming and outgoing flights.

These numbers are a bit different if we consider the route network. In this case the fraction of routes between two ECTL airports ranges between 69% and 73%, while the fraction of routes in the database between two non ECTL airports ranges between 2.4% and 2.8%.

## V. NETWORK METRICS: A REVIEW

As mentioned above we will consider airport networks, i.e. graphs where the nodes (vertices) are the airports and there exists a link between two airports if there is at least a flight that connects them. The links may (*flight network*) or may not (*route network*) be directed. They will always be weighted, according to the number of flights between the two airports.

Several metrics can be considered in order to characterize a network. We will hereafter introduce some of them:

- *Degree* - The degree of each node in a network is given by the number  $k$  of links of the node. In the case of directed networks one can estimate the degree  $k_{in}$  (estimating the number of links incoming to the node) and the degree  $k_{out}$  (estimating the number of links out-coming from the node). For the route network, the degree of an airport is the number of other airports that can be reached from it in a day, i.e. the number of destinations. The vertex degree distribution  $P(k)$  is one of the key tools we may use to characterize the network configuration [10], since this function determines the way nodes are connected. Specifically, a crucial role is played by the way the distribution decays for large degree values. For example, when the distribution decays like a power-law  $P(k) \propto k^{-\alpha}$ , it means that the probability of finding airports with a very large number of flight connections is much higher than what expected in the case of a binomial distribution of degrees, usually associated to a random graph [15]. Such networks are sometimes termed *scale free networks*.

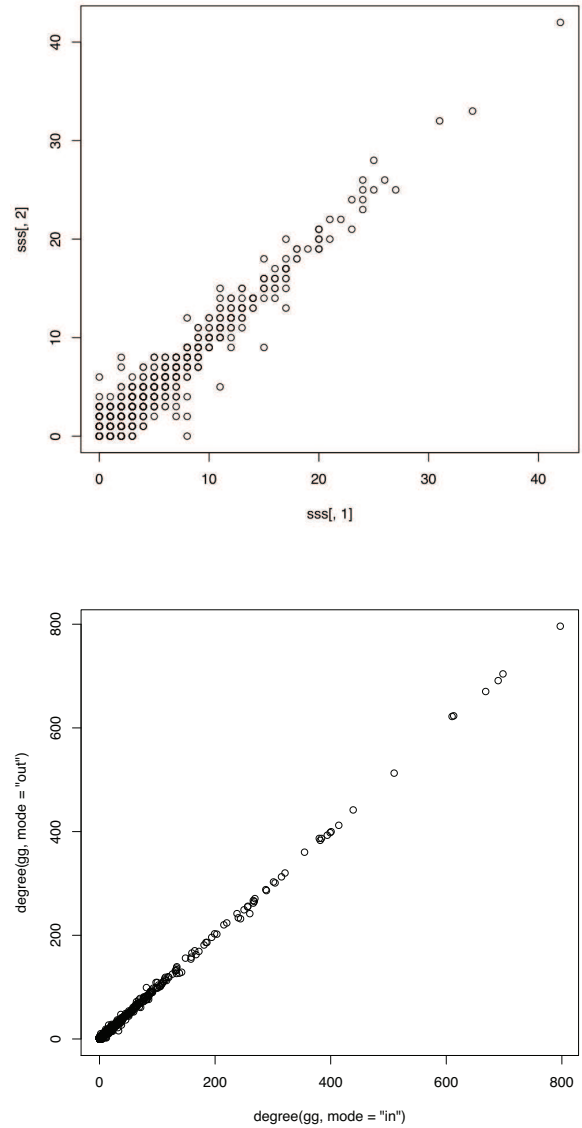


Fig. 1. **Upper Panel.** Number of flights from A to B vs. number of flights from B to A for each pair  $(A, B)$  of connected airports. **Lower Panel.** Number of outgoing vs number of incoming flights for each airport.

- *Weight* - As mentioned above, each link can be assigned a number quantifying the level of connection between the two connected nodes. This is the weight of the link. In our case, as mentioned above the weight is given by the number of flights existing between the two airports in the given period of time. Similarly to the previous case, the link weight distribution  $W(k)$  is one of the key tools we may use to point out the network topology.
- *Strength* - The node strength is simply the sum of the weights associated to the links that originate from (terminate in) it. In our case, the strength of a node gives the total number of flights departing from (arriving to) an

airport. In the case of the route network, the strength is given by  $n_{AB} + n_{BA}$ .

- *Average Path Length* - Let us call  $\ell_{ij}$  the shortest distance between vertex  $i$  and vertex  $j$ . This is defined as the minimum number of links that connects any two nodes  $i$  and  $j$  in the considered network. The Average Path Length  $\langle \ell \rangle$  is then defined as:

$$\langle \ell \rangle = \frac{1}{N(N-1)} \sum_{i,j} \ell_{ij} \quad (1)$$

where  $N$  is the number of nodes in the network. In our case  $\langle \ell_{ij} \rangle$  measures how many flights are needed in average to reach airport  $j$  starting from airport  $i$ . The Average Path Length  $\langle \ell \rangle$  is therefore a measure of how well connected the airports are.

- *Diameter* - The Diameter of a network is defined as the maximal value of  $\ell_{ij}$ :

$$D = \max_{i,j} \ell_{ij} \quad (2)$$

In our case the diameter of the network gives the maximal number of flights needed to reach any two airports in the network.

- *Betweenness* - The node betweenness  $B_k$  gives a measure of the relative importance of a node in a network. It is defined as:

$$B_k = \sum_{i,j \neq k} \frac{\ell_{ij}(k)}{\ell_{ij}} \quad (3)$$

where the sum is extended over nodes  $i$  and  $j$  that are both different from node  $k$ . Here  $\ell_{ij}(k)$  is the minimum number of links that connects any two nodes  $i$  and  $j$  and passing through vertex  $k$ . In our case  $\ell_{ij}(k)$  is the minimum number of flights through airport  $k$  needed to reach airport  $j$  starting from airport  $i$ . This is a measure of how central is airport  $k$  in the network. High values of  $B_k$  indicate that such airport is reached by many travellers moving from one airport to another and thus probably it behaves like a hub.

- *Clusters* - Clusters in a network are sets of nodes that are only connected within themselves and are not connected with any other node outside the cluster. In our case, if airports A, B, and C belong to the same cluster, it means that there exists no flight that connects one of them starting from any other airport D in the network. The existence of clusters of airports would indicate that there are regions that can not be reached from outside. A network where all vertices belong to the same cluster is referred to as a *connected network*.

## VI. STATISTICAL CHARACTERIZATION OF AIR TRAFFIC NETWORKS

In this section we present some statistical characterization of the flight network and of the route network. As we have discussed above we can consider different networks depending on whether we want to include only ECTL airports or all the airports in the database.

Table I shows some summary statistics of the networks when one includes all the nodes (airports). We consider one network per day in order to estimate the statistical fluctuations in time. The table shows for each day the number of nodes (airports), the number of flights, and the number of links in the route network. This last number is clearly the number of routes, where route from A to B and route from B to A are counted only once. The next metric is the average strength,  $\langle s \rangle$ , in the route network. This corresponds to the average number of flights (incoming and outgoing) for an airport in the network. The table shows that on average there are 45 flights taking off or landing in an airport per day. The system is however very heterogeneous. In fact Table I shows that the maximal value of the strength,  $\max(s)$ , is more than one thousand. In order to have a full characterization of the distribution of strength (flights) per airport, we show the cumulative probability of the strength  $s$  for one day in the left panel of figure 2. The distribution is quite fat tailed and not compatible with an exponential tail.

A different way of characterizing the topological properties of the network is by considering the degree  $k_A$  of a node (airport) A, i.e. the number of airports connected with a direct flight with A (in the considered day). Table I shows the average degree  $\langle k \rangle$ . Table shows that on average an airport of the database is connected with approximately 12 destinations. Again the system is very heterogeneous. In fact the maximum degree,  $\max(k)$ , is more than 200. In the right panel of figure 2 we show the cumulative probability of the degree  $k$  for one day. Also in this case we observe a large heterogeneity, but in this case the tail of the distribution is well fit by an exponential function.

We consider the relation between the degree (number of destinations) and the strength (number of flights) of an airport. In figure 3 we show the relation between the two variables in a log-log scale. In the region of degree  $k > 20$  we fit a power law relation  $s = C k^\beta$ . The best fit gives the value  $\beta = 1.39$  indicating a superlinear relation between number of flights and number of destinations. This means that if an airport doubles the number of destinations, it typically increases the number of flights by a factor  $2^{1.39} \simeq 2.45$ .

The considered networks are not completely connected. In fact, direct inspection reveals that the system is partitioned in several clusters of airports. However, as Table I shows, the largest cluster covers more than 97% of the airports. The remaining clusters are made of very few airports. Therefore the network has a giant component essentially covering the entire system.

Finally, we consider the average path length connecting two airports. The path length  $\ell_{AB}$  is the minimum number of flights needed to reach airport B from airport A. The average path length  $\langle \ell \rangle$  is the mean value of  $\ell$  across all airport pairs. By taking into account the directionality of flights (i.e. by considering the flight network) we find that the average path length  $\langle \ell \rangle_s$  is a little bit larger than 3 (see Table I). This is intuitive since typically one reaches an hub from a local airport, then uses a flight connecting two hubs, and finally

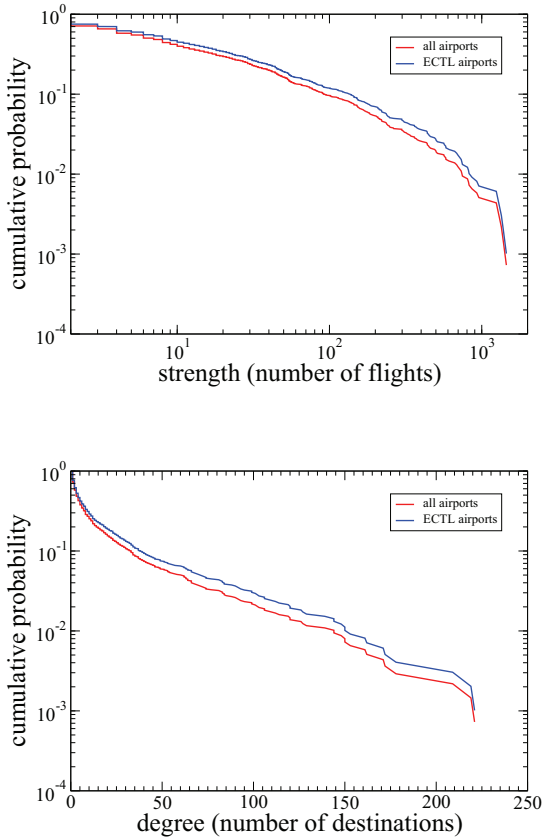


Fig. 2. **Upper Panel:** Cumulative density function of strength  $s$  (left, in log-log scale) for the whole set and for only ECTL airports. **Lower Panel:** Cumulative density function of degree  $k$  (right, in semi log scale) for the whole set and for only ECTL airports.

reaches the final destination. By neglecting the directionality of links, i.e. by using the route network, one obtains the average path length  $\langle \ell \rangle_k$  shown in Table I. This is by definition smaller than  $\langle \ell \rangle_s$ . However the table shows that these numbers are not dramatically different. The reason is that, as discussed above, the vast majority of airport pairs are connected in both directions in a typical day, therefore the directionality of links does not change significantly the path length.

We also consider the properties of links. We have seen that in the route network a link is characterized by its weight, i.e. the number of flights in the route (in one day). The mean value of the weight is  $\langle w \rangle = 3.70$ , i.e. a route has typically 3.7 flights per day. Note however that the standard deviation is 5.1. and the maximum is  $\max(w) = 95$ , indicating again a large heterogeneity. In figure 4 we plot in semi log scale the cumulative distribution of link weight. The tail of the distribution is well approximated by an exponential function.

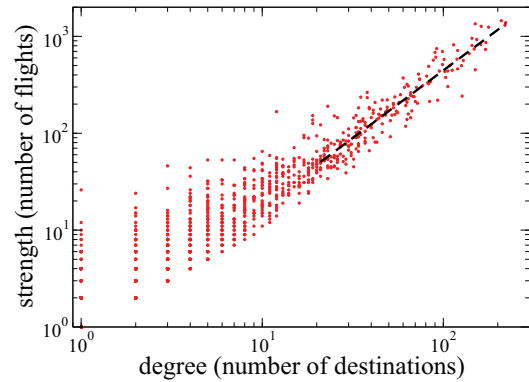


Fig. 3. All airports. The dashed line is a best fit with a power law in the region of degree larger than 20. The estimated exponent is 1.39. Similar results are observed for ECTL airports only.

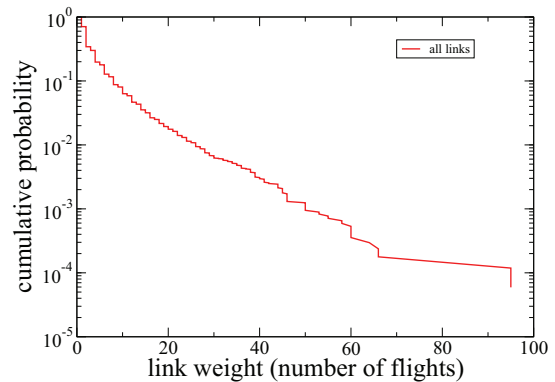


Fig. 4. Cumulative density function of link weight  $w$ , i.e. number of flights per route. The plot is in semi log scale.

## VII. PRELIMINARY ANALYSIS OF DELAYS

We define delays as the difference of landing time in  $m3$  file and landing time in  $m1$  file. We recall that  $m1$  files have information about the last filled flight plan whereas  $m3$  files have information about updates of the last filled flight plan obtained from radar data.

The fraction of flights with more than 15 minutes of delay ranges between 15% and 28%. These numbers are consistent with the fraction reported in the PRR report 2010 (25%). The fraction of flights with more than one hour of delay ranges between 0.34% and 2.6%.

Figure 5 shows the probability density function of delays in semi log scale for each day of the first week of June 2011. The central part is roughly described by a Laplace distribution (asymmetric). Note also that there is a relatively large fraction of flights with a *negative* delay.

Finally, we consider whether some simple topological prop-



day	nodes	flights	links	$\langle s \rangle$	$\langle k \rangle$	$\max(s)$	$\max(k)$	largest cluster	$\langle \ell \rangle_s$	$\langle \ell \rangle_k$
W June 1	1375	31224	8498	45.4	12.4	1452	221	1356	3.34	3.29
Th June 2	1296	27660	7757	42.6	12.0	1426	226	1264	3.33	3.27
F June 3	1291	28280	8039	43.8	12.5	1384	219	1256	3.31	3.23
Sa June 4	1153	25289	7642	43.8	13.3	1343	245	1126	3.17	3.15
Su June 5	1188	28175	8104	47.4	13.6	1451	232	1167	3.14	3.12
M June 6	1342	30435	8243	45.3	12.3	1494	221	1327	3.35	3.29
T June 7	1348	30397	8131	45.1	12.1	1494	220	1313	3.34	3.28

TABLE I  
FULL NETWORK.  $\ell$  IS THE PATH LENGTH. FOR A DETAILED EXPLANATION, SEE TEXT.

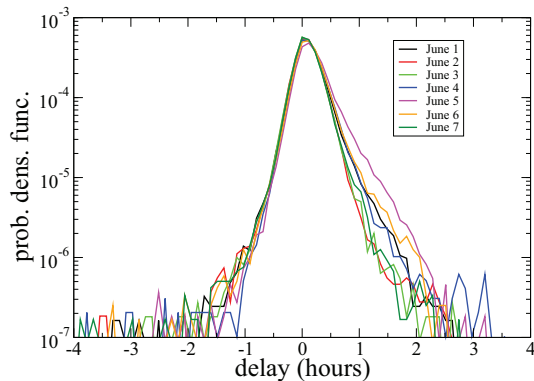


Fig. 5. Probability density function of delays in the first week of June 2011. The y-axis is in log scale.

erties of the airport network is related to flight delay. As a preliminary result, in Figure 6 we show the relation between the fraction of delayed flights (with a threshold of 15 minutes) and the strength of the node, i.e. the number of flights arriving or departing in an airport in a given day. To have a higher statistic we pool together all days in the week June 1-7, 2011. We observe that airports corresponding to nodes with large strength tend to have an higher fraction of delayed flights. A linear regression between the two variables restricting to airports with more than 300 flights in a day gives a noisy but statistically significant relation ( $R^2 \simeq 0.1$ ). This result indicates that airports with a higher traffic tend to have a larger fraction of delayed flights.

### VIII. PERSPECTIVE WORK

The results presented in this paper are relative to a preliminary analysis of the flight and route networks. Further analyses performed in the future activity of the ELSA project will include a complete study of the flight delays as a function of the above mentioned network metrics. For example it is expected to be of particular interest the way the fraction of delayed flights depends on the network average path lengths. Furthermore, we plan to study how characteristics of flight and route networks subsets affect the whole system. More precisely we will consider regional and airline sub-networks, as well as temporal layers of the networks (e.g. peak and off-peak) and

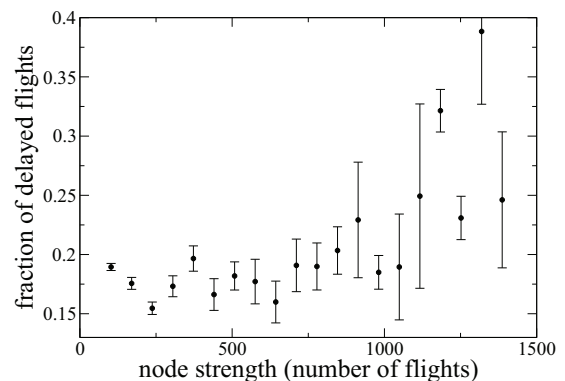


Fig. 6. Fraction of delayed flights (more than 15 minutes) as a function of the node strength (number of flights in a day) for the whole week June 1-7, 2011. Data are binned on the x axis. Error bars are standard errors.

will study how the delays in these subsets differ from the original network. A particular attention will be paid to the network properties of hub airports and the relative fraction of delayed flights. We plan to analyse whether secondary airports directly linked with an hub are able to absorb the delays generated by an hub during the rush hours or if the delays are propagated to the secondary airports. This point is especially relevant for the transition to the target SESAR scenario, as capacity gains are expected to come from a greater use of uncongested secondary airports [16].

Finally, we are planning to study traffic delays at the level of flight segments in order to characterize delays in terms of the segment typology. For example, we might investigate what are the stylized facts of delays in segments close to airport areas or in segments that are far away from airports and nevertheless experience high traffic volumes

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